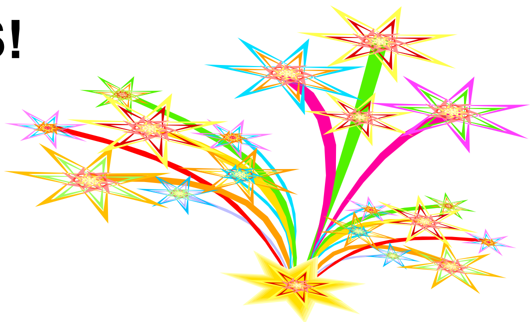
CHAPTER 1: FIREWORKS!



To teachers:

We know you can *do* it, that you can get the *answers* to the school problems!
But that is not enough! A mathematics educator must be able to reflect, at the *meta-level*, on the *processes* and *products*, and be able to communicate with others about it!

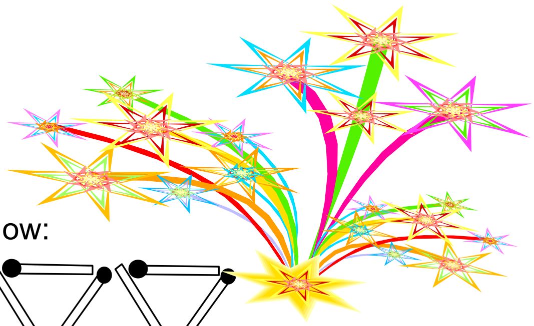
<u>First *your* mathematics . . .</u>
Discuss the patterns, relationships and processes in the situation . . .
Share your *methods* with each other.

<u>Now analyse the *mathematics* . . .</u>
Is *this* algebra? Where is the *algebra? What* is the algebra?
What are the *meanings* and *use* of variables and formulae here? (*Are* here variables and formulae?)

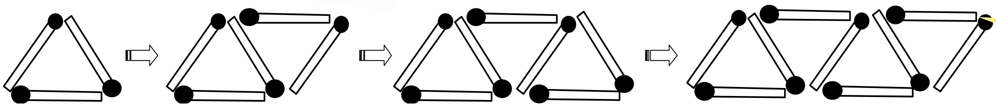
<u>And about *learning* mathematics . . .</u>
Can *your* pupils solve these problems?? Can you *anticipate* their strategies?
Try to formulate explicitly: What *worthwhile* can pupils productively learn from such activities?

<u>About *teaching* mathematics . . .</u>
Be explicit: Where, why and how can you use such activities in your teaching?

FIREWORKS!



1. Sylvia forms **triangle** patterns with matches as shown below:

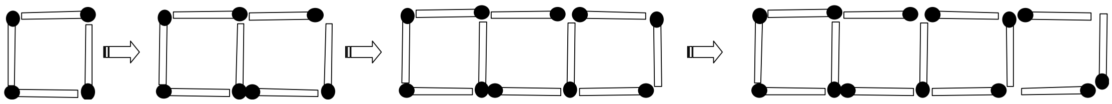


(a) Complete the following table. Describe any number patterns!

# triangles	1	2	3	4	5	6	7	8	9	10	100
# matches	3	5	7	9							

(b) Sylvia wants to know how many matches she needs *in total* to build *the whole sequence*, up to 100, e.g. to build the first 5 patterns she needs 3 + 5 + 7 + 9 + 11 = 35 matches.

2. On another day Sylvia forms **square** patterns with matches as shown below:

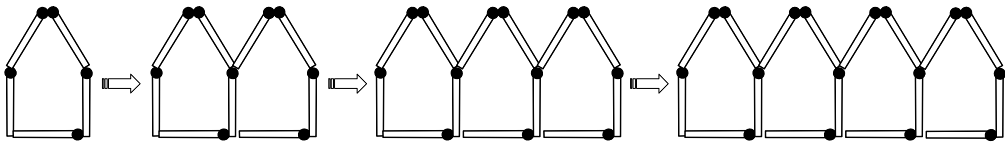


(a) Complete the following table. Describe any number patterns!

# squares	1	2	3	4	5	6	7	8	9	10	100
# matches	4	7	10	13							

(b) How many matches does Sylvia need in total to build the whole sequence up to 100?

3. On another day Sylvia forms **pentagon** patterns with matches as shown below:

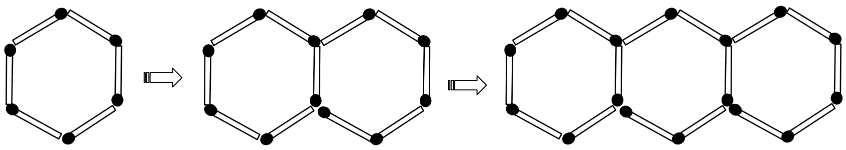


(a) Complete the following table. Describe any number patterns!

# pentagons	1	2	3	4	5	6	7	8	9	10	100
# matches	5	9									

(b) How many matches does Sylvia need in total to build the whole sequence up to 100?

4. On another day Sylvia forms **hexagon** patterns with matches as shown below:



How many matches does Sylvia need to build 100 **hexagons**?

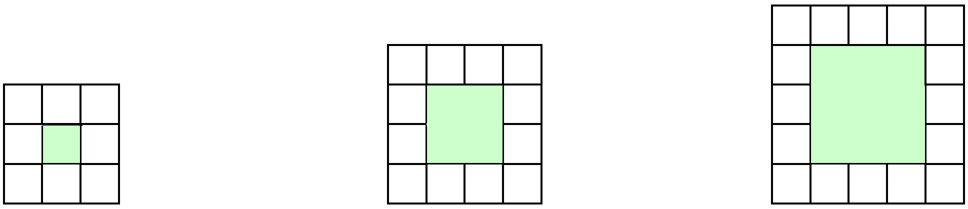
5. What is the *same* and what is *different* in the situations in 1, 2, 3 and 4?

6. How many matches will Sylvia need to build 100 **decagons** (a polygon with 10 sides)?

7. How many matches will Sylvia need to build 100 n**-gons** (a polygon with n sides)?

Flower beds

The town council of *Tendele* decides to beautify the town. They build square flower beds of different sizes and surround them with square tiles as shown in these sketches:



1 by 1 bed   2 by 2 bed   3 by 3 bed

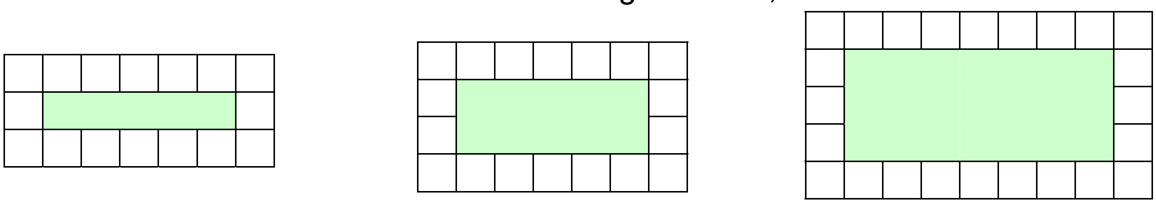
1. How many tiles are needed for a
   (a) 10 by 10 bed   (b) 50 by 50 bed   (c) 100 by 100 bed?

2. Construct a formula giving the number of tiles for an n by n bed.

3. John uses the formula $2(n + 2) + 2n$
   Jane uses the formula $4(n + 1)$
   Joan uses the formula $4(n + 2) - 4$
   Javo uses the formula $4n + 4$
   Judy uses the formula $(n + 2)^2 - n^2$

   Who is correct? *How do you know?*
   Can you *explain* (show!) each student's thinking to get the formula?

4. The town council decides to also build rectangular beds, as shown below:

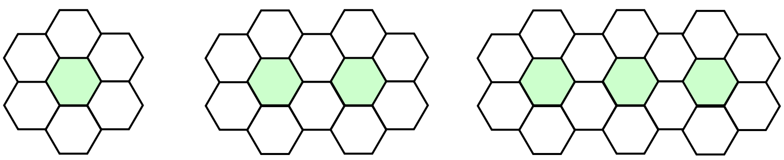


5 by 1 bed   5 by 2 bed   7 by 3 bed

How many tiles are needed for a
(a) 6 by 5 bed   (b) 100 by 90 bed?   (c) m by n bed?

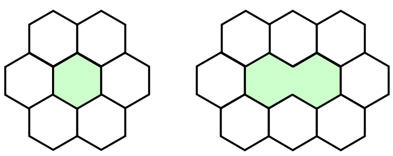
5. The city council wants to use hexagonal paving tiles to build flower beds according to the different designs shown in the sketches below.
   In each case, how many tiles do the council need for a 100-bed? And for a n-bed?

Design 1



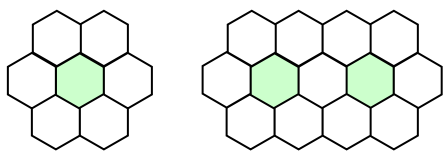
1-bed   2-bed   3-bed

Design 2



1-bed   2-bed

Design 3



1-bed   2-bed

FIREWORKS NOTES

We briefly describe different approaches and different representations that characterise the nature of algebra, specifically the idea of *generality*, the idea of a *model*, the meaning of *algebraic expressions* and the meaning of *equivalent transformations* . . .

It is useful to distinguish two different underlying *thinking strategies (processes)*, which I will call *numerical pattern recognition (induction)* and *structural analysis (deduction).* The processes are not necessarily distinct – there is often an interplay from one to the other. Both processes are important in doing and learning mathematics.

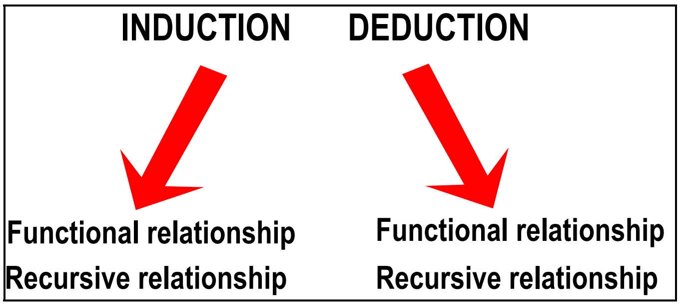
The mathematical relationship between two variables can always be described in terms of either

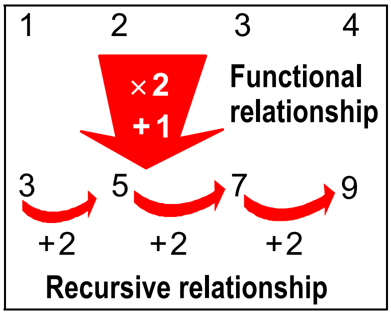
- the *functional relationship* between the variables which can lead to a formula such as, here, $m = \mathrm{f}(t) = 2t + 1$, or
- the *recursive relationship* between (successive) function values leading to a formula, as here, $\mathrm{f}(t + 1) = \mathrm{f}(t) + 2$.

Both types of relationships are important in (the learning of) mathematics. They emphasise different aspects of the relationship:

- a *functional formula* such as $m = 2t + 1$ makes it easy to find function values, solve equations, ...
- a *recursion formula* such as $\mathrm{f}(t + 1) = \mathrm{f}(t) + 2$ underlies the study of sequences and series, and the important concepts of change, rate of change, gradient and derivative.

Both induction and deduction can lead to a functional or a recursive relationship.



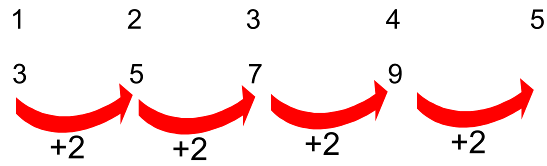


Numerical pattern recognition (induction)
The process of induction consists of two sub-processes:

1. pattern recognition in a finite set of data (abstraction)
2. pattern extension to cases not in the present set (generalisation)

One can focus on the *numbers* given in the table (we call this the *database)* and recognise a vertical (functional) relationship $m = 2t + 1$ which easily yields all the solutions. Or one can recognise a horizontal (recursive) pattern, which can serve as a model to generate additional information about the situation.

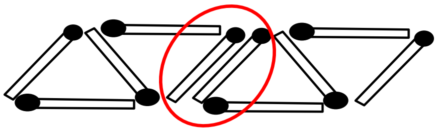
The nature of a *model* is that it *simulates* the physical situation, so that one can generate information by manipulating the mathematical model instead of the practical situation. Therefore, even the simple recursive pattern f(t + 1) = f(t) + 2



is a useful model that allows us to determine the number of matches needed for 5, 6, 7, ... triangles *without having to physically build or draw the triangles and count the matches*. (This is what we mean with *predict:* to use a mathematical *model*, *not the physical situation* or a physical model of the situation, to generate additional, unknown information about the situation.) However, it is not initially feasible to continue this recursive pattern until t = 100.

Efforts at generalising

Most learners recognise the need for *a better method*. However, few learners seem to use the functional relationship – most learners try to adapt the recursive relationship, but this often leads to errors, e.g. f(10) = 21, so f(100) = 10 × f(10) = 210. However, this is not a valid property of f. It can be *disproved* by a simple special case – a *counter example* – e.g. f(4) ≠2 × f(2), or analysing the physical situation and realising that we will be repeating some matches[1].



2 ×f(2) = 2 ×5 = 10.
But f(4) = 9 in the database

The property that f(kx) = k × f(x) is a property only of the proportionality model f(x) = mx.

When children make this mistake here, it is very important that the teacher should help them to compare this situation with situations like in the attached Matches activity, and analyse how the situations are the same and how they are different, i.e. they should analyse the *properties* of the functions.

A valid "shortcut" can be found by looking at the horizontal pattern with different eyes, i.e. not working with the function *values*, but with the *structure* of the values:

f(1) = 3 = 3
f(2) = 5 = 3 + 2 = 3 + 1×2
f(3) = 7 = 3 + 2 + 2 = 3 + 2×2
f(4) = 9 = 3 + 3×2
f(5) = 11 = 3 + 4×2

We can recognise the pattern in this *structure* and generalise it to

f(t) = = 3 + (t–1)×2 .... (1)

So, then, if t = 100: f(100) = = 3 + 99×2
= 201

We may notice that this is indeed a special case of the formula $T_n = a + (n-1)d$ for an arithmetic series, so it is an important base for further learning.

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[1] Once we realise the *cause* for the error, we can adapt the method to give correct results, e.g. in the sketch f(4) = 2×5 = 10, is wrong, but realising the overlap, we can correct it: f(4) = 2×5 – 1 = 9. Similarly, f(100) = 10×f(10) – 9 = 201.

Structural analysis (deduction)

While the above came directly from looking at the *numbers* and ignoring the *matches* (picture), some learners focus on the *process* of packing the *matches* and ignore the numbers. They easily formulate "you start off with 3 matches and then add another 2 matches for every additional triangle that you build". This is a model in the form of *words* (the equivalent to (1) in *symbols*) and also easily yields

$$f(100) = 3 + 99 \times 2 = 201$$

Syntactic meaning of algebraic expressions

It is important to note that the models all describe *a computational procedure*, e.g.

- in words: take the *number* of triangles, multiply it by 2 and then add one
- as a flow-diagram: —[**×2**]—[**+1**]→
- as an algebraic expression using symbols: $2 \times t + 1$

It is vitally important to develop meaning by fostering connections between different representations of the model. We should help children to reflect on the connections, for example to connect the *constant difference in the table*, with the parameter (gradient) a in the formula $y = ax + b$.

All the examples share the property that there is a constant difference. We should develop and reflect on *The Difference Theorem*:

- If there is a constant difference (of a), then the formula is linear ($y = ax + b$).
- If the formula is linear ($y = ax + b$), then there is a constant difference (of a).

Equivalent transformations

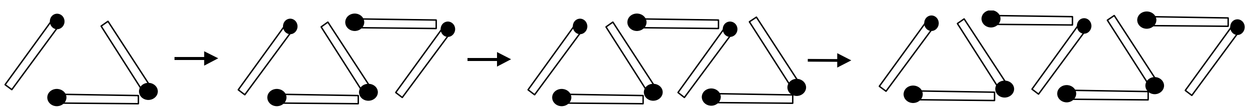
It is interesting to compare the recursive formula $3 + (t - 1) \times 2$ with the functional formula $2 \times t + 1$. Of course they yield the same *value* for the same value of *t*. *So they are simply different computational procedures!* Different *methods!* If we now convince ourselves that they are "the same" at the formal/deductive level, when we say

$$\begin{aligned} & 3 + (t - 1) \times 2 \\ &= 3 + 2t - 2 \\ &= 2t + 1 \end{aligned}$$

we are <u>not</u> saying that 2t + 1 is the "answer" of 3 + 2(t – 1)!! We are merely saying that they are different procedures (methods) to calculate the same thing, therefore they give the same numerical result for the same value of the variable. That is what equivalent transformation (i.e. algebraic manipulation) means in this context!

Semantic interpretation of algebraic expressions

The recursers know exactly what their formula $m = 3 + 2 \times (t - 1)$ means: you start with 3 matches for the first triangle, and then you add an extra 2 matches for every extra triangle that you make. But what does the functional relationship $m = 2t + 1$ mean in the physical situation? Well, it is as simple as this:



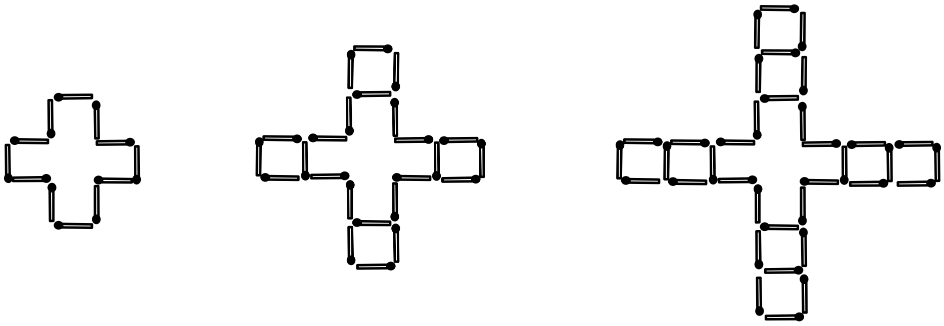
F(1) = 2 + 1   F(2) = 2×2 + 1   F(3) = 3×2 + 1   F(4) = 4×2 + 1

Is a picture worth a thousand words?

Can you use these new "eyes" to *see* the formulae in the sketches for squares, pentagons, ...?

MATCHES

1. Thandi builds crosses with matches like this:



Picture 1 Picture 2 Picture 3

The table shows the number of matches she uses for different pictures:

Picture number	1	2	3	4	5	6	…	20	…	60	….	100
Number of matches	12	24	36	48								

(a) Complete the table.

(b) Explain to some of your classmates how you got to your answer for Picture 60.

(c) Below Saul, Siketla, Mandy and Vuyo use different *methods* (plans) to calculate the number of matches in Picture 60.
Who will get the right answer? *Explain why.*
If someone will *not* get the right answer, *explain why not.*
Which of these plans do you prefer? Explain *why*.

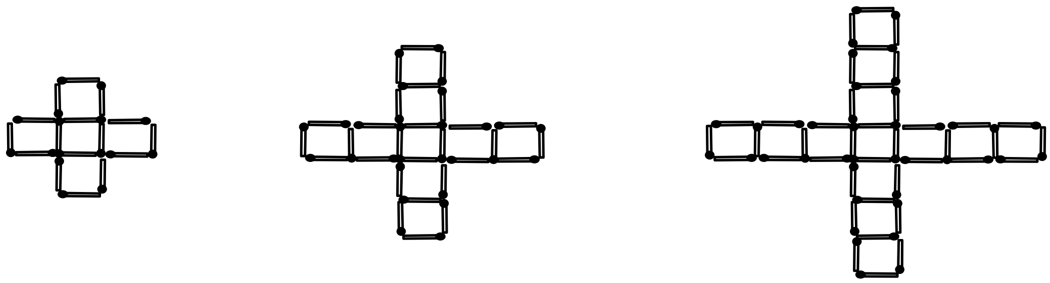
Saul's plan:
I see from 12 to 24 is +12, and from 24 to 36 is +12, and from 36 to 48 is also +12. So I continue to add 12 until I reach Picture 60.

Siketla's plan:
I know $6 \times 10 = 60$. So I take the number of matches I got for Picture 6, which is 72 and I times it by 10 to get the number of matches in Picture 60: 72×10 matches.

Mandy's plan:
I multiply the Picture number by 12. So in Picture 60 there are 60×12 matches.

Vuyo's plan:
In Picture 1 there are 12 matches. Then you add another 12 matches for every picture you make. For Picture 60 I make another 59 pictures, so I must add another 59×12 matches. So in Picture 60 there are $12 + 59 \times 12$ matches.

2. On another day Thandi builds crosses with matches like this:



Picture 1 Picture 2 Picture 3

The table shows the number of matches she uses for different pictures:

Picture number	1	2	3	4	5	6	…	20	…	60	….	100
Number of matches	16	28	40	52								

(a) Complete the table. Explain your method to calculate the number of matches for Picture 60.

(b) Saul, Siketla, Mandy and Vuyo use different *methods* (plans) to calculate the number of matches in Picture 60. Who will get the right answer? Explain *why* or *why not.*

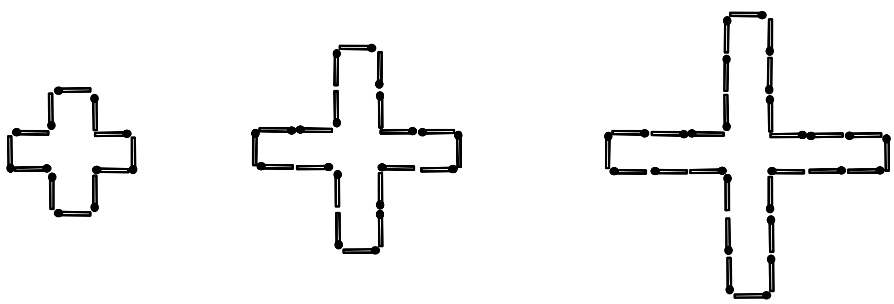
Saul's plan:
I see from 16 to 28 is +12, and from 28 to 40 is +12, and from 40 to 52 is +12.
So I continue to add 12 until I reach Picture 60.

Siketla's plan:
I know 6 × 10 = 60. So I take the number of matches I got for Picture 6 and I multiply it by 10 to get the number of matches in Picture 60.

Mandy's plan:
I multiply the Picture number by 12, and then I add 4.
So in Picture 60 there are 60 × 12 + 4 matches.

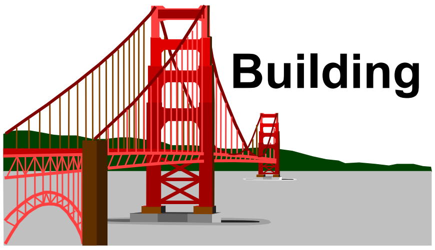
Vuyo's plan:
I see from 16 to 28 is +12, and from 28 to 40 is +12, and from 40 to 52 is +12. So Picture 6 has 76 matches. Now from 6 to 60 means I must add 54 × 12 and then I must still add the number of matches for Picture 6. So it is 76 + 54 × 12.

3. On another day Thandi builds crosses with matches like this:

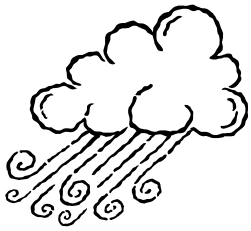


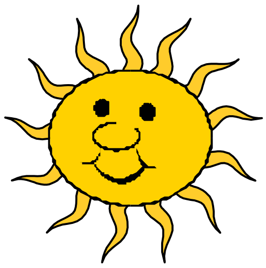
Picture 1 Picture 2 Picture 3

Describe Saul, Siketla, Mandy and Vuyo's metod for these crosses.
Whose method gives the right answer for Picture 60? Explain *why* or *why not.*

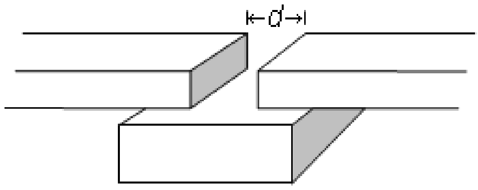


Building bridges 1





When building a bridge or railway line, engineers have to leave small gaps between sections to allow for heat expansion. This gap should not be too small (why not?), but also not too large (why not?).



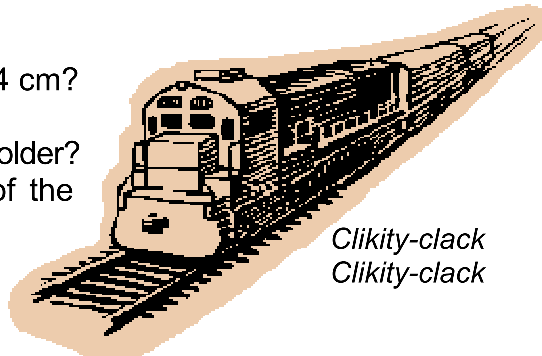
For a certain bridge the size of the gap is 2 cm at a temperature of 0 °C. For each 1 °C that the temperature rises, the gap becomes smaller by 0,05 cm.

1. Complete the following table showing the size of the gap at different temperatures. Explain your *method*.

Temperature (t °C)	0	1	2	3	4	5	10	20	30
Gap size (d cm)									

2. On the day they built the bridge the temperature was 24 °C. What size should the engineers make the gap?

3. At what temperature will the size of the gap be 1,14 cm?



4. What happens to the size of the gap as it becomes colder? What if it becomes *very* cold? What will the size of the gap be at a temperature of $^{-}5$ °C?

5. What happens to the size of the gap as it becomes hotter? What if it becomes *very* hot? What will the size of the gap be at a temperature of 50 °C?

6. If they have to build a similar bridge in the desert they have a major problem, because the temperature can range from $^{-}10$ °C to 55 °C! *Why is it a problem?* *Advise the engineers what to do! Be specific, use numbers and substantiate your advice!*

7. On the graph paper overleaf, draw a graph of the size of the gap against temperature. *Interpret* the situations in questions 1-6 in the graph.

NOTES: BUILDING BRIDGES 1

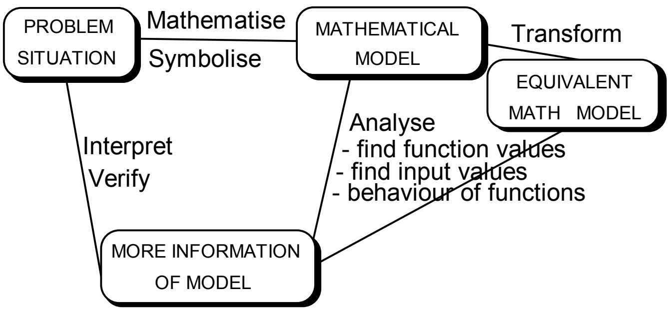
This is an interesting situation, illustrating the power of mathematics to *model* a “real-life” situation.

Functions

It is a typical *functional situation*: We have two variables where the one variable (the size of the gap, d cm) is *dependent* on the other variable (the temperature, t°C). To understand the nature of the relationship between the variables, we must understand the scientific principle that an increase in temperature leads to an expansion of the metal, and therefore to a *decrease* in the size of the gap. It is therefore a *decreasing function*. (Note that in the *real world*, the friction of e.g. a heavy train at high speed can increase temperature of the rail tremendously, so in a real-real problem the independent variable should be the temperature of the rail, not the air temperature!)

Different representations of functions

The function is here given in *words* and can be transformed to a *table*, a *formula* or a *graph*, depending on which representation is more convenient for a particular purpose. These transformations are important skills in learning algebra, as shown below:



From...to	Words	Table	Graph	Formula
Words				
Table				
Graph				
Formula				

The words and the table show that there is a *constant rate of change* of 0,05 cm/°C. It is this *constant rate of change* (gradient) that defines it as a *linear function*, with a straight line *graph* and *formula* of the form y = mx + c.

The questions can all be solved *numerically*, directly from the word formulation or the table representation. But the advantages of a formula should be clear!

The activity involves the following function *problem types*:

- Finding function values, i.e. given t, find $d(t)$.
- Finding input values, i.e. solving equations, e.g. given $d(t)$, find t.
- Working with the behaviour of function values (a decreasing function, rate of change, domain and range).
- Finding a formula.

Semantics and syntactics

One aspect to understand is the *syntactical meaning* of the expression: the expression is a shorthand notation telling us exactly which operations to carry out on which numbers in which order. So $d = 2 - 0{,}05t$ tells us that to calculate the value of d for any value of t, we should multiply the value of t by 0,05 and then subtract the result from 2.

The other aspect to understand is the *semantic meaning* of the formula, the *meanings* it derives from its *physical referents*, e.g. t is not just an abstract number, it is the temperature; d is not just a number, it is the size of the gap.

The power of mathematics is immense, and at all stages symbols make a major contribution to this power. But without the ability of the mathematician to invest them with meaning, they are useless. Richard Skemp, 1971, 89

Physical referents

This activity creates an opportunity to reflect on the *connections* between the physical situation and the models in the form of a formula, table and graph. For example, we "see" the physical situation in the formula $d = 2 - 0{,}05t$: At $t = 0^oC$ the size of the gap is 2 cm, and we know that as t increases the size of the gap becomes *physically* smaller; *mathematically*, we *subtract* a positive amount from 2, making the gap numerically smaller. However, if the temperature *decreases* below $0\,^oC$, we know, physically, that the gap becomes larger, and the mathematics must behave accordingly, e.g. for $2 - 0{,}05(^-1)$ to become larger, it will have to be $2 + 0{,}05$ ("a minus times a minus is a plus"), illustrating again that *we construct mathematics to fit reality*!

A further essential aspect of the sematic meaning of the formula is to understand the *physical referents* for the different parts in the formula! So it is important that we know that in $d = 2 - 0{,}05t$ the "2" is the gap size at 0^oC and the "0,05" is the gradient or rate of change – how fast the size of the gap changes as the temperature changes. Furthermore, this rate of change is specific to the specific bridge material – it is the so called *linear expansion coefficient* of the material, which is unique to each substance, like density.

Parameters

Therefore, to solve the desert problem, we should be able to change these "constants" to *parameters,* e.g. $d(t) = d(0) - kt$ where $d(t)$ is the gap size at $t\ ^oC$, $d(0)$ is the gap size at $0\ ^oC$ and k is the expansion coefficient. These two parameters can be varied to suit the conditions: If we make $d(0)$ larger, then $d(0) = 2{,}8$ cm will guarantee that the gap does not close up before $56\ ^oC$. However, note that if $d(0) = 2{,}8$ cm, then $d(-10) = 3{,}3$ cm, which is probably too large! Why is that? The other option is to specify a different material for the rail with a *larger* expansion coefficient, at least k = 0,36 cm/^oC. Why? An interactive Excel file[2] is available, dynamically illustrating the effects of changing the parameters on the graph of the model, and therefore illustrating both the physical problem and its solution. In essence we are here dealing with the *transformation of functions*: $f(x) + k$ as a vertical shift, and $kf(x)$ as stretching and shrinking which is for a straight line interpreted as a change in the gradient.

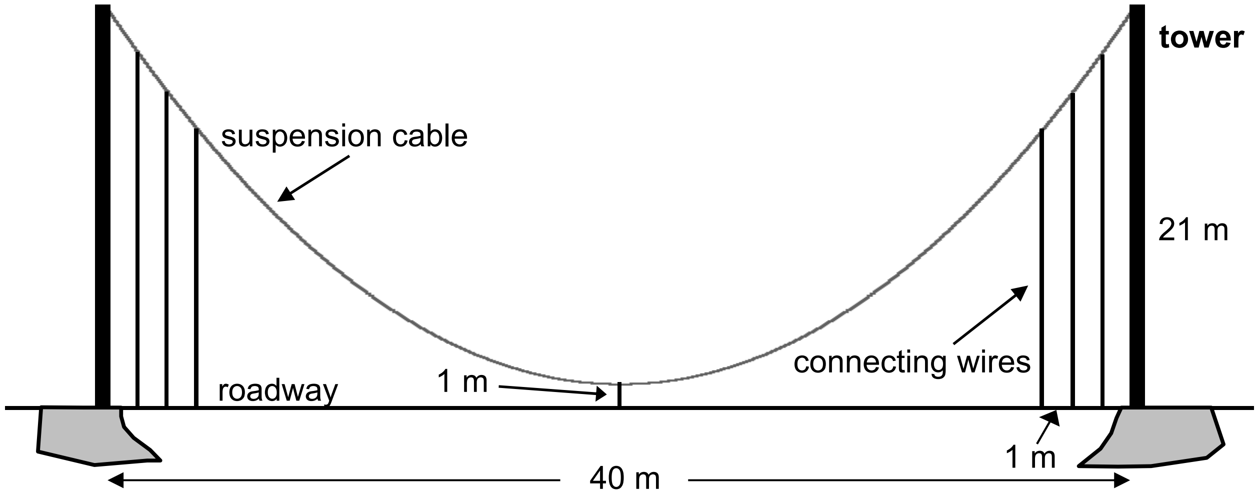
Domain and range

The *domain* and *range* of the function also are very important here: Although there are no restrictions on the values of the variables in the *abstract* model $d = 2 - 0{,}05t$, the domain and range are determined by the *physical* situation: d cannot be less than zero. Therefore $d \geq 0$ and therefore $t \leq 40$ (*this is the desert problem!*) and this should also be reflected and interpreted in the graph and in our interpretations of the mathematical solutions to the practical problem.

We can therefore use the function (graph or formula) to *extrapolate* (extend beyond the known values) to find unknown values, but should do so with care. This is like *inductive generalisation*, the pitfall of induction, that the pattern is not necessarily valid beyond the known *database*, because the structure does not continue. Here, the structure holds only for the *physical domain and range*.

BUILDING BRIDGES 2

Engineers must build a hanging bridge over a river. The roadway of the bridge will be hung from a suspension cable with connecting wires, 1 m apart. At its lowest point, the connecting wire is 1 m long. The suspension cable is supported by two towers at the ends, which are each 21 m high and are 40 m apart.



It is essential that the lengths of the connecting wires should be *exact*, otherwise the bridge is unsafe! The lengths cannot be found by *practical measurement while* building the bridge!

This is how engineers do it: They find a *formula* for the *shape* of the suspension cable, then they use this formula as a *model* to *calculate* the lengths of all the connecting wires *beforehand*.

Make a table giving the length of each of the connecting wires.

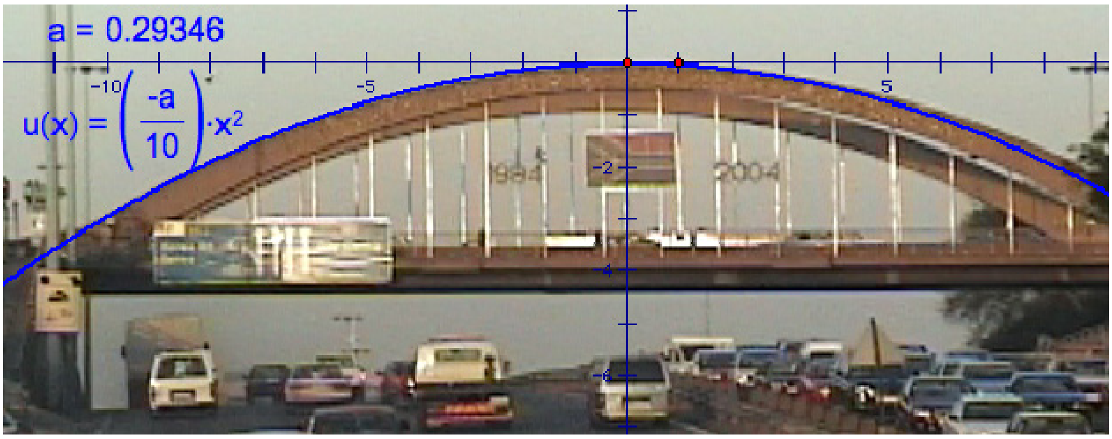
The famous Golden Gate hanging bridge in San Francisco, USA

NOTES: BUILDING BRIDGES 2

Mathematical modeling means that we replace the real-world objects with mathematical objects, and then manipulate the mathematical objects instead of the physical objects to gain more information about the physical situation.

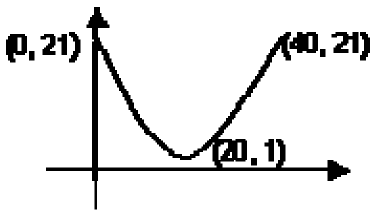
In our bridges example, we *imagine*, we *impose* a coordinate system on the physical situation (it is in our *head*, it is not part of the real world) as a *mathematical lens* to help us *mathematise* the situation – introducing mathematical concepts, notations, measurements as numbers ... and then manipulate the concepts, symbols, numbers as a mathematical model to analyse the situation. This was the great invention of Descartes in 1834, leading to "coordinate geometry" where geometric curves could be represented with algebraic formulae. This was a major invention in the history of mankind! So tremendous was the contribution of Descartes that he is called the father of Mathematics and the father of philosophy..

The *choice* of position of the system of axes influences the *form* of the formula and the *complexity* of the calculations!



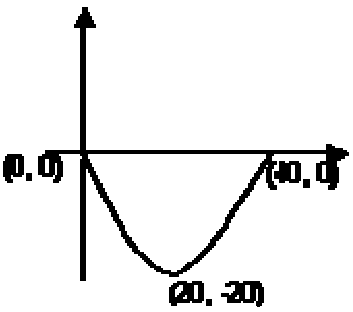
Here are 4 different positions of the axes, with the corresponding formulas:

a.

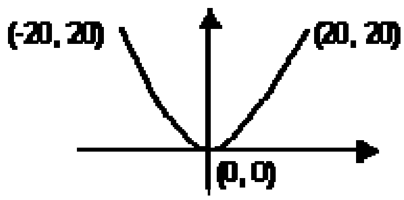


$y = 0{,}05(x - 20)^2 + 1$

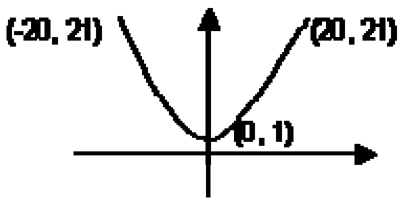
b.



$y = 0{,}05x(x - 40)$
or
$y = 0{,}05(x - 20)^2 - 20$

c. 

$y = 0{,}05x^2$

d. 

$y = 0{,}05x^2 + 1$

The formulae are different, but *equivalent*. The lengths of the wires correspond to the y -values of (d), so maybe it is the easiest, because you do not need extra calculations for the lengths!

This is how we obtained the function for (d):

The formula is of the form $y = ax^2 + c$ (1)

The point (0, 1) lies on the graph (it is the y -intercept), so it satisfies equation (1).

So c = 1

The point (20, 21) lies on the graph, so it satisfies equation (1):

So 21 = 400a + 1, so a = 0, 05

Once you have a formula, it is easy to *calculate function values*, but you must still *interpret* the context in terms of your chosen system. For example, to solve problem 1:

If you use the (a) system, "15 m from the centre" means x = 5 or 35, which should of course give the same answer because of symmetry:

$y(15) = 0{,}05(5 – 20)^2 + 1 = 0{,}05(35 – 20)^2 + 1$

In the case of (d) x = 15:

$y\ (15) = 0{,}05 \times 15^2 + 1 = 12{,}25$ m

Chapter 2: Mathematics All Around Us

In this chapter you will experience the power of modeling as a mathematical tool to describe situations in real life and as an analytical tool to gain additional information about the situation.

The activities in this unit focus on input and output relationships given in words, tables and flow-diagrams. You will deal with activities for which you can describe the relationship between the input and output (i.e. find a rule between the input and output) and activities for which there are no rules.

You will explore pattern-generating activities in which information could be provided in words, numerically or in the form of a picture of the situation. These activities will give you opportunities and practise in expressing generality (i.e. seeing ***samenesses*** *in different situations).*

You will encounter and be assessed on the following problem types

- *finding input values*
- *finding output values*
- *finding the relationship between the input and output value (constructing a formual).*

Activity 1: The shopping mall

1. Mario sells doughnuts at a stall in the shopping mall. He does not want to make calculations every time so he started preparing the following table to help him.

Number of doughnuts	1	2	3	4	5	6	7	8	9	10	…
Total cost (in cents)	29	58	87	116	145						

   (a) Complete Mario's table.

   (b) How much would 25 doughnuts cost?

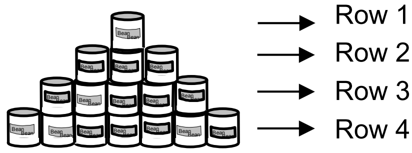
   (c) Describe your rule for calculating the cost of any number of doughnuts.

   (d) How do you know that your rule is correct?

   (e) A customer pays Mario R3,48. How many doughnuts did she buy?

2. Every year the Mall organises a quiz for children. Answer the following two questions that appeared in the quiz.

   (a) In a can–stacking competition competitors must stack the cans as shown in the picture. Lester is sure that he can win the competition if he is able to stack 100 rows of cans. How many cans will he need? Explain how you know that you are correct.



   (b)

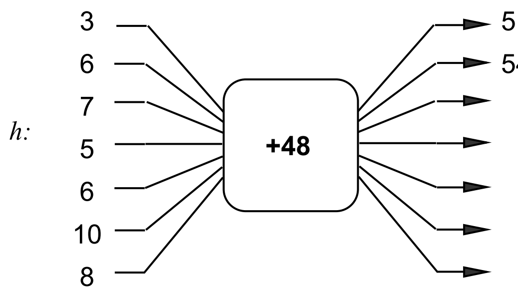
Picture 1 Picture 2 Picture 3 Picture 4

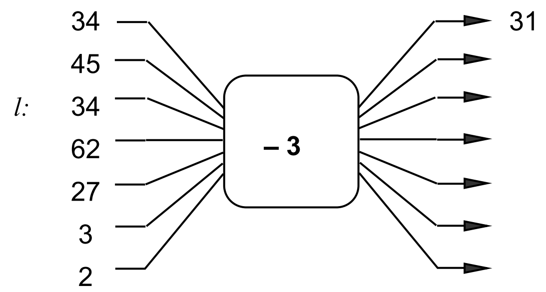
   If the pattern above is continued, how many squares will Picture 53 have? Explain!

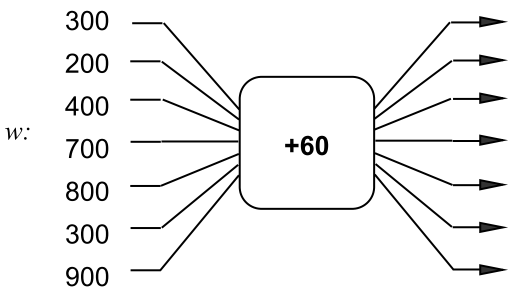
3. Compare your answers and methods for 1 and 2 with some classmates.

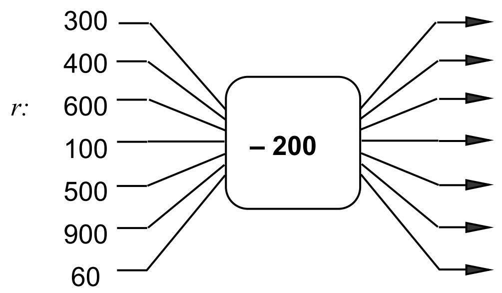
Activity 2: Flow diagrams

1. Complete the flow diagrams.









2. The numbers on the left of a flow diagram are called the *input numbers*, and the numbers on the right are called *output numbers.* In *h* above, 54 is the output number that corresponds to the input number 6.

   (a) In *l* above, what is the output number for the input number 34?

   (b) What is the input number in *w* above that corresponds to the output number 760?

   (c) In which flow diagram above does the output number 260 correspond to the input number 200?

   (d) In which flow diagram above does the output number 27 correspond to the input number 30?

   (e) What will the output number in *h* above be if the input number is 30?

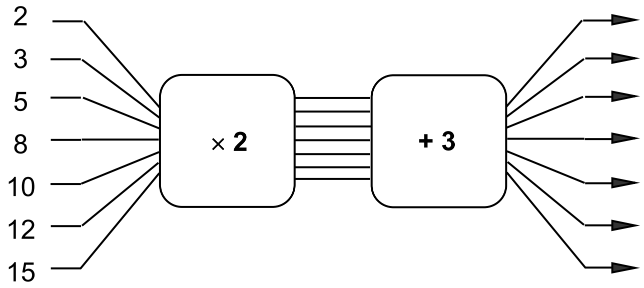
   (f) What will the input number in *h* above be if the output number is 148?

   (g) The output number for *r* is ⁻100 ("minus 100" or "negative 100") if the input number is 100. What will the output number for *r* be if the input number is 40?

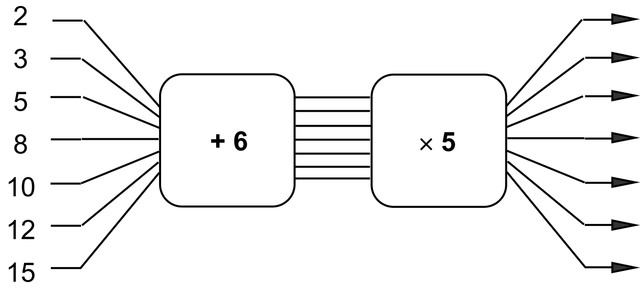
   (h) What will the output number in *l* be if the input number is 0?

3. Complete the flow diagrams.

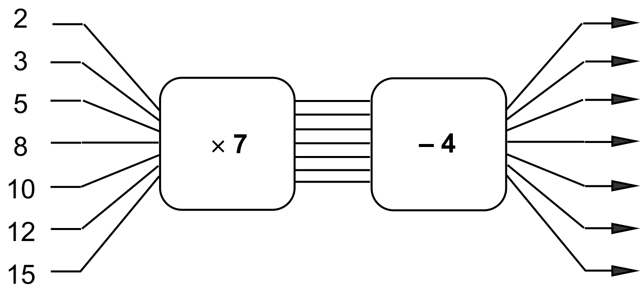
f:



g:



h:



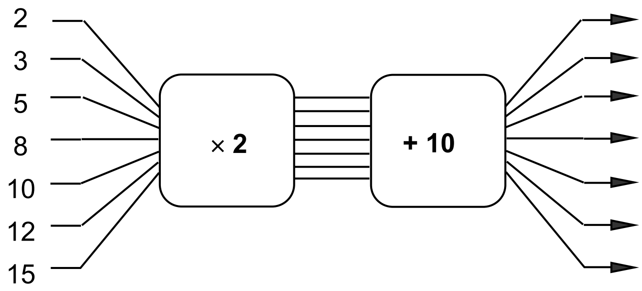
4. Each of the three flow diagrams above has two instructions. For instance *f* has the instructions [× 2] and [+ 3]. Such instructions are also called *operators.*

   (a) What are the operators in *g* above?

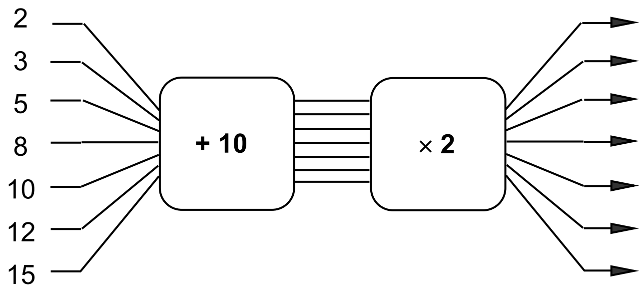
   (b) What is the second operator in *h* above?

5. Complete the flow diagrams:

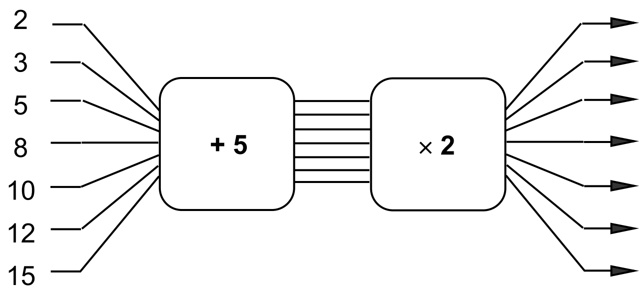
k:



l:



m:



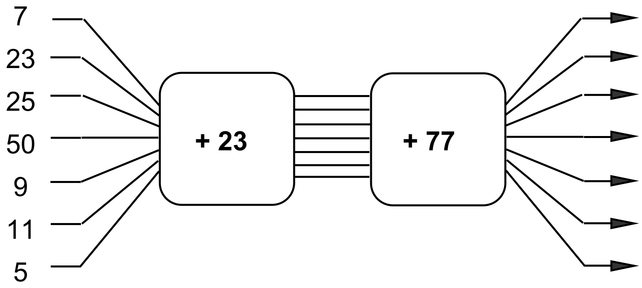
6. Note that *k* and *l* have the same operators, but in different orders. For the same input numbers *k* and *l* produce different output numbers. The order (sequence) of the operators make a difference in this case.

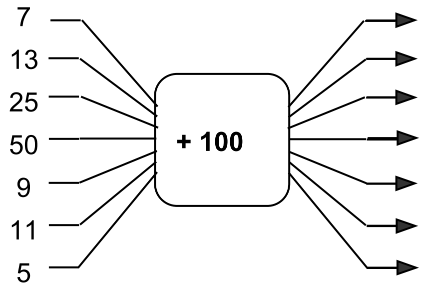
   (a) Do you think the order of the operators will always make a difference? Explain.

   (b) Also note that *k* and *m* above produce the same output numbers, although one of the operators differ. We say that these two flow diagrams are *equivalent* (they produce the *same* results although the operators are *different*).

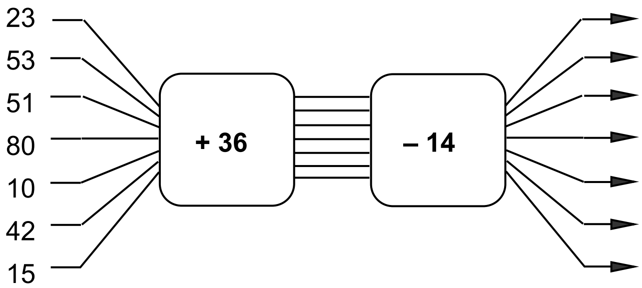
7. Complete the flow diagrams.

f:

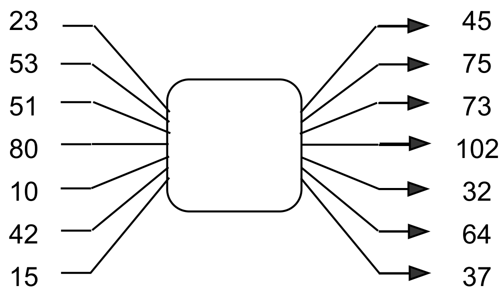




k:



In this flow diagram you have to fill in the missing operator:



Activity 3: Travelling and communicating

1. The cost of hiring a bus to travel from Johannesburg to Pietersburg and back is R750.
   (a) If 30 people go on the trip, what must each passenger pay if they share the cost equally?
   (b) If 15 people go on the trip, what must each passenger pay if they share the cost equally?
   (c) Complete the table:

Number of passengers	5	10	15	20	30	40	50	n
Cost for one passenger								

   (d) Write a formula with which the cost for one passenger can be calculated, for any number of passengers travelling on the bus.

2. The cost of hiring a bus to travel from Johannesburg to Durban and back is R1200, plus an additional R20 for each passenger.
   (a) Write a formula with which the cost for one passenger can be calculated, for any number of passengers travelling on the bus.
   (b) Complete the table:

Number of passengers	8	10	16	20	25	32	40
Cost for one passenger							

3. Jill has written the following formula for the situation in question 2:

   Cost per passenger = 1200 ÷ *number of passengers* + 20

   Lester uses Jill's formula to calculate the cost per passenger if there are 60 passengers. So he first writes:

   Cost per passenger = 1200 ÷ 60 + 20

   He then calculates as follows:

   60 + 20 = 80

   1200 ÷ 80 = 15, so the cost per passenger is R15.

   But now Lester is puzzled. He thought each passenger has to pay at least R20.
   Can you help him?

4. The following table of monthly rates is supplied by a telephone company.

# call units	50	100	150	200	300	350	400	450	500	550
Account in rands	43,50	51,00	58,50	66,00	81,00	88,50	96,00	103,50	111,00	118,50

(a) What is the charge per call unit?

(b) What is the monthly rental for a telephone, if no calls are made during that month?

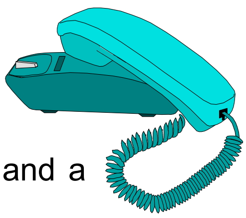
(c) Make a flow diagram with which the account for one month can be calculated for any number of call units.

(d) Make a formula with which the account for one month can be calculated for any number of call units.

5. Telephone company A charges a monthly rental of R36 and 13c per call unit. Complete the following table.

# of call units	50	51	100	104	150	180	200	220	250	290	300
Account in rands	42,50		49,00								

6. Draw a flow diagram to show what calculations one has to do to complete the table in question 5.



7. Telephone company B charges a monthly rental of R45 and a charge of 8c per call unit. Complete the following table.

# of call units	50	150	200	250	300	350	400	450	500	550	600
Account in rands											

8. The cost of a long distance train ticket is calculated as follows:

R12,40 for any journey, plus 24c per kilometre.

(a) What will a ticket for a journey of 1346 km cost?

(b) If the ticket costs R220,24, how far is the journey?

9. (a) How much does it cost to have a telephone in a house?

(b) How much does it cost to make a journey by train?

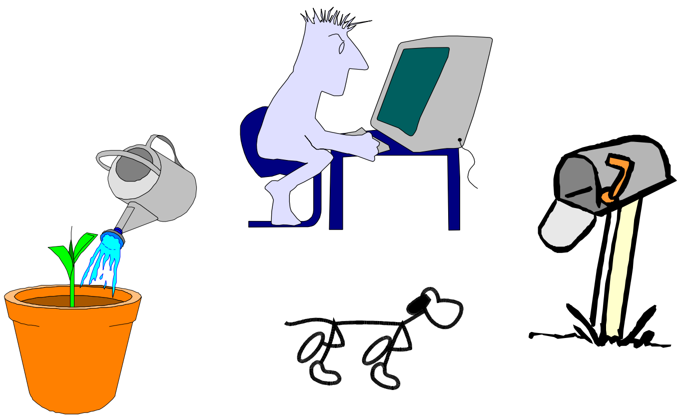
Activity 4: Business

Joe decided to run his own business during his summer vacations. When neighbours went on vacation, Joe would water their plants, collect their mail, walk their dog, etc. He charged a fee of R10 per household, plus R2 for each hour of work.

1. Joe filled in this table after each job so that he could tell how much his customers owed him. Calculate how much money each customer paid him.

Name of customer	Time spent working (in hours)	Cost to customer (in rands)
	T	R
Mr Harris	1	
Jenny	2,5	
The Ramsey's	7	
Mrs Ames	3,25	
The Heath's	15,2	
Rachael	11	

2. Write a rule that would show how much Joe earned from any customer. Discuss your rule with a few classmates.
3. Mr Scullard paid Joe R20. How many hours did he work for Mr Scullard?
4. Calculate Joe's *total* income for the vacation. Can you use two different methods to calculate the total? Describe your methods in words.



Activity 5: Car hire

Two car rental companies advertise as follows:

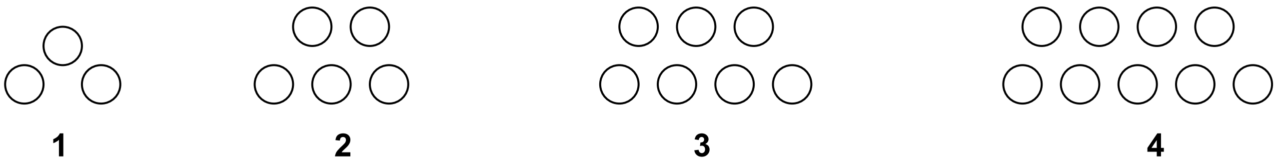
***Axis* cars are cheapest!**
Only R48 per day plus 75c per kilometre
for a clean luxurious car.
Contact our agents at 011-7536722.
Please book early to avoid disappointment.

Drive a Hearts car soon.
60c per kilometre plus R60 per day.
Available at all airports and stations.
Watch for our signs.

1. What does it cost to hire a car from Axis for 3 days, and to travel 387 km?
2. What does it cost to hire a car from Axis for 1 day, and to travel 387 km?
3. What does it cost to hire a car from Hearts for 1 day, and to travel 387 km?
4. What does it cost to hire a car from Axis for 1 day, and to travel 37 km?
5. What does it cost to hire a car from Hearts for 1 day, and to travel 37 km?
6. Which company is cheaper, Axis or Hearts?

Activity 6

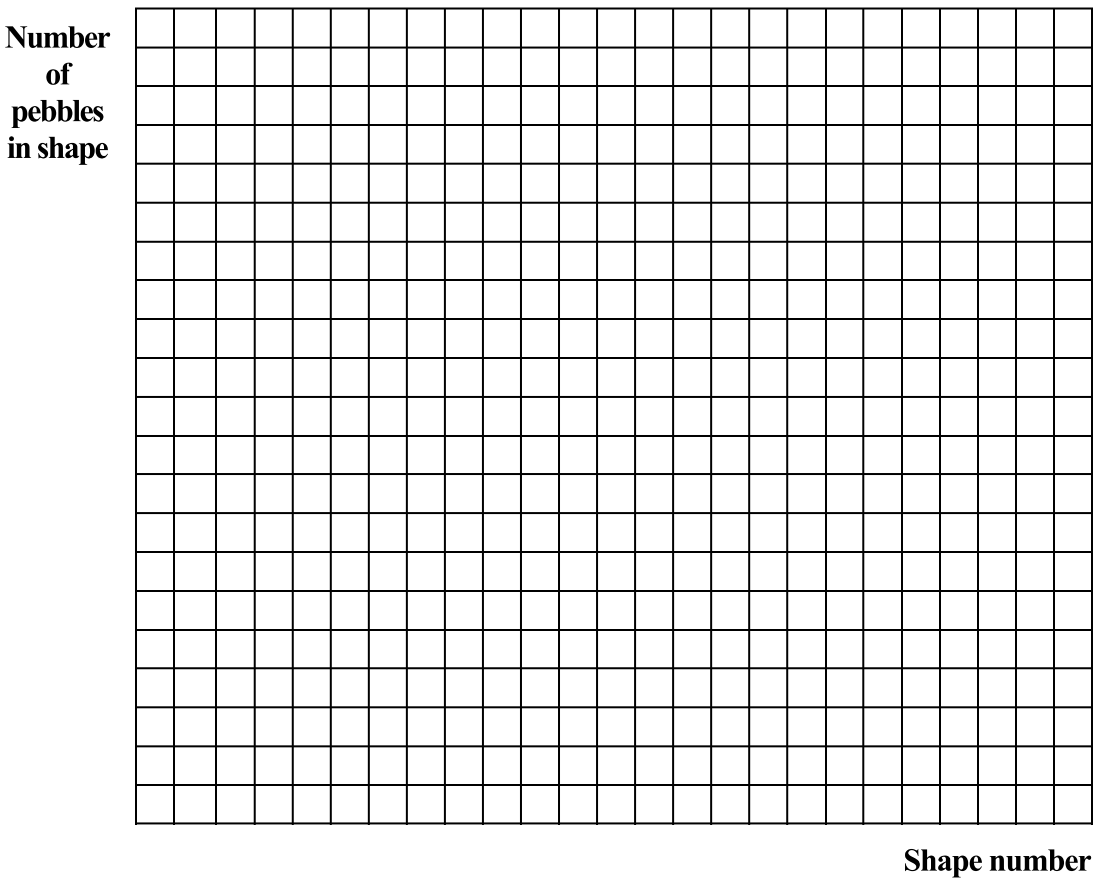
Pebbles …



1. Complete the table.

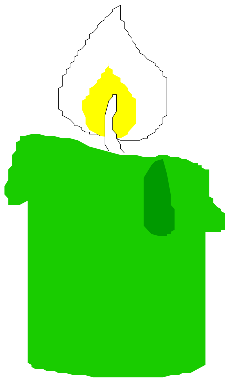
Shape number	1	2	3	4	5	6	7		10		30		60		*n*
Number of pebbles in shape	3	5	7	9											

2. Make a graph of your findings.



Activity 7: Candles

the manufacturer of a new type of candle claims that their candles burn very long.
To test the claim, the science class did the following experiment: they lit four different candles and measured their lengths every hour for five hours. Their results are in the following tables.

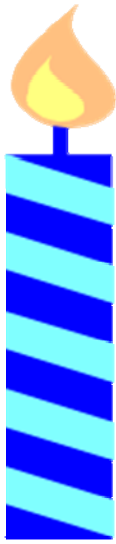


Candle 1

Time (hours)	0	1	2	3	4	5
Length (cm)	36	34,5	33	31,5	30	28.5

Candle 2

Time (hours)	0	1	2	3	4	5
Length (cm)	16	15,5	15	14,5	14	13,5



Candle 3

Time (hours)	0	1	2	3	4	5
Length (cm)		11,6	11,2	10,8		10

Candle 4

Time (hours)	0	1	2	3	4	5
Length (cm)	46	44	42	40	38	36

Now help the science class with their conclusions:

1. They forgot to fill in some information for Candle 3. Complete it for them.

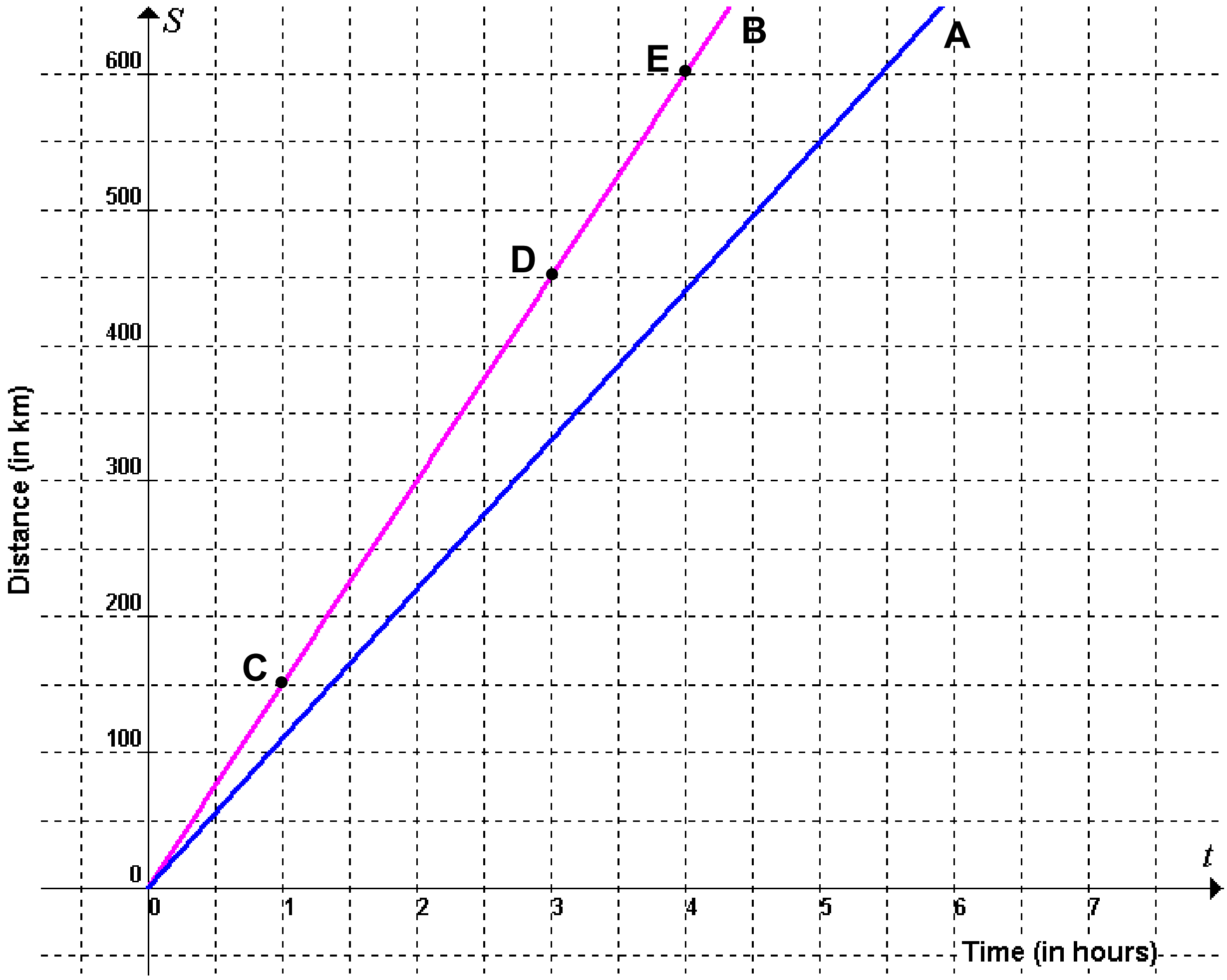
2. Which candle will burn the longest? How long? *Explain!*

3. After how many hours will Candle 1 be 10 cm long?
   After how many hours will Candle 2 be 10 cm long?
   After how many hours will Candle 3 be 10 cm long?
   After how many hours will Candle 4 be 10 cm long?

4. Draw graphs on the same system of axes overleaf showing how the lengths of the four candles change as the time changes.

5. Interpret the situations in questions 1-3 in the graph.

Activity 8: Speed



*Two motorists A and B each traveled at a constant speed. The distances (*S *km) they covered after* t *hours are represented in the graphs above.*

1. Use the graphs to make a table of time-distance values for each motorist.

2. What is the speed of motorist B between points
   (a) C and D
   (b) C and E
   (c) D and E
   Explain your answers. What do you notice?

3. What is the speed of motorist A?

4. Write an algebraic formula that represents the relationship between time and distance for each motorist.

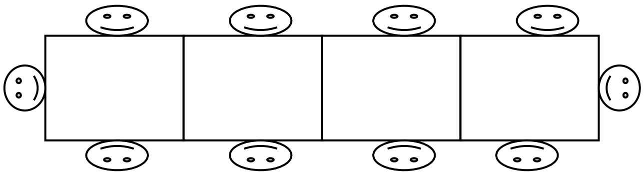
5. Which part of the algebraic *formulae* represents the speed of each motorist?

6. Explain how you can tell from the *tables* at which speed each motorist is driving.

7. Explain how you can tell from the *graphs* at which speed each motorist is driving.

Activity 9: Party

1. Lester has a party planned for 13 July. He arranges small square tables next to each other in a straight row, so that one person sits on each of the available sides of the table. So for example with 4 tables, he can seat 10 people, as shown in the sketch below.



   (a) If Lester arranges 25 tables as shown above, how many people can be seated?

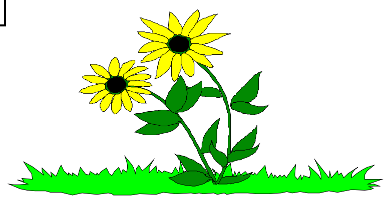
   (b) How many tables must he pack next to each other if he wants to seat 74 people. Explain!

   (c) How many tables must he pack next to each other if he wants to seat 47 people. Explain!

   (d) Write in words how Lester would calculate the number of tables required to seat *n* people. How can you convince yourself, your group and your teacher that you are correct?

2. The growth of a seedling was measured by the science class over a two-week period. The following information was recorded.

Day	0	2	4	6	8	10	12	14
Height (mm)	0	3	6	9	12	15	18	21



   (a) What was the daily growth of the seedling?

   (b) When was the seedling 10,5 mm tall?

   (c) After 11 days what was the height of the seedling?

   (d) Explain how the age and the height of the seedling are related.

   (e) If the plant continued to grow at the same rate when would it have been 60 mm high?

   (f) When the seedling is **D** days old, what would its height, **H**, be?

   (g) Do you think the seedling will continue to grow at this rate? Explain your answer.

Chapter 7: Algebraic Language

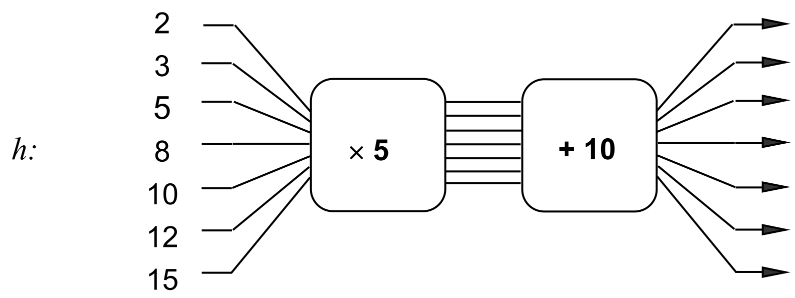
In this chapter you will extend your understanding of the meaning of symbols in algebra (what is x?) and how to use these symbols in the algebraic language.

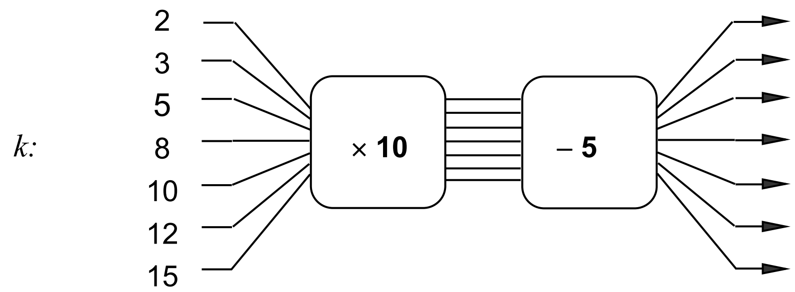
You should be able to:

- *Translate from a flow diagram to an algebraic expression and vica versa*
- *Translate from words to an algebraic expression and vica versa*
- *Find values of an algebraic expression by substituting in an algebraic expression*
- *Read corresponding values from a table*
- *Use brackets and exponential notation*

Activity 1: Writing and reading formulas

1. (a) Complete the two flow diagrams.





(b) The working of flow diagram h can be described in words by the expression
multiply the input variable by 5 and then add 10
or simply "multiply 5 and then add 10".
Describe the working of flow diagram k with such an expression.

If the letter symbol x is used to indicate “the input variable”, then the working of flow diagram h can be described as follows (this is called an **algebraic expression**):

$$5 \times x + 10$$

(c) Describe the working of flow diagram k with an algebraic expression.

2. In each case write an algebraic expression for the given verbal expression, using the letter symbol x to represent the input variable.
   (a) multiply the input number by 15 and add 8
   (b) multiply the input number by 20 and subtract 35

3. In each case write a verbal expression for the given algebraic expression.
   (a) $20 \times x + 50$
   (b) $10 \times x + 15$
   (c) $15 + 10 \times x$
   (d) $20 - 3 \times x$
   (e) $3 \times x - 20$

4. (a) Do the expressions in 3(b) and 3(c) give the same results?
   (b) Do the expressions in 3(d) and 3(e) give the same results?
   To answer the above questions, you may find it useful to complete the following tables:

Input variable	1	2	3	4	5	6	7
$10 \times x + 15$							
$15 + 10 \times x$							
$20 - 3 \times x$							
$3 \times x - 20$							

In algebraic language, the × -sign is usually omitted.

So we write $3x$ instead of $x \times 3$ or $3 \times x$.

When a variable is to be multiplied by a constant like in $3x$, the constant is called the **coefficient**. In a case like $2 - 5x$, the coefficient of x is $^{-}5$.

We also write $3(x + 5)$ instead of $3 \times (x + 5)$.

5. Write each of the following expressions in "normal" algebraic language, i.e. without using ×-signs, and by putting coefficients on the left of symbols representing variables.
   (a) $17 \times x + 15$
   (b) $y \times 2{,}4 - 14$
   (c) $(z \times 7 + 5) \times 8$
   (d) $27 - x \times 13 + (14 - x \times 3) \times x$

6. What is the coefficient of x in each of the following expressions?
   (a) $3x - 5$
   (b) $2x + 3$
   (c) $1{,}7x + 3$
   (d) $10 - 2x$

Each of the expressions in question 6 specifies how the output values of a function can be calculated for any given input value represented by x.

Activity 2: More algebraic language

1. The output numbers (values) of the function *f* can be calculated with the formula

   output number = 3 × *input number* + 15

   (a) Complete the following table for *f.*

Input variable	2	5	10	15	21	28	51
Value of *f*							

   (b) Which of the following verbal expressions describe the above function correctly?

   add 15 then multiply by 3

   multiply by 3 then add 15

   (c) Write algebraic expressions for the two verbal expressions in (b).

2. The output numbers for the function *g* can be calculated with the formula $3(x + 15)$, where x indicates the input variable.

   (a) Which of the following verbal expressions describe this function correctly?

   add 15 then multiply by 3

   multiply by 3 then add 15

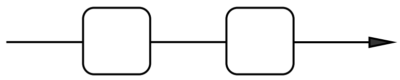
   (b) What is the difference between $3x + 15$ and $3(x + 15)$?

   (c) What do the brackets mean?

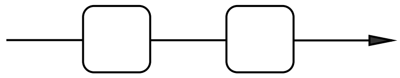
3 (a) Does $3x + 15$ and $15 + 3x$ mean the same or different things in algebraic language.

   (b) Does $3(x + 15)$ and $(15 + x) \times 3$ mean the same or different things in algebraic language.

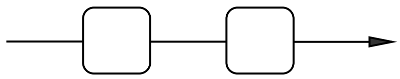
4. Fill in operators so that the flow diagram will produce the same results as the algebraic expression $3x + 15$.



5. Fill in operators so that the flow diagram will produce the same results as the algebraic expression $3(x + 5)$.

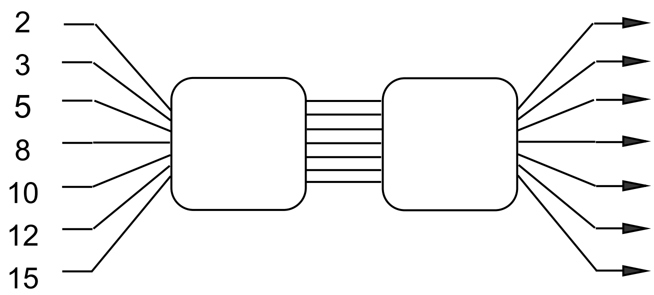


6. Fill in operators so that the flow diagram will produce the same results as the algebraic expression $3(x + 15)$.

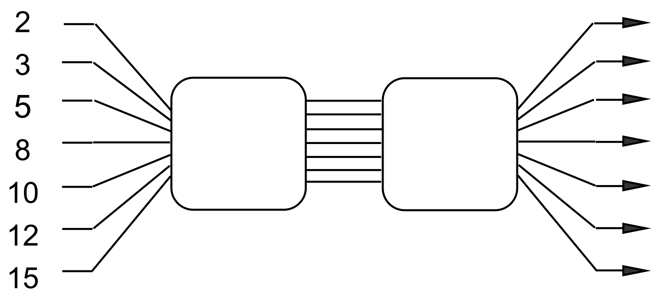


7. The function h is described by $5x + 20$, the function k by $5(x + 20)$, and the function m by $5(x + 4)$. Complete the following flow diagrams. Also fill in the operators. What do you notice? Discuss.

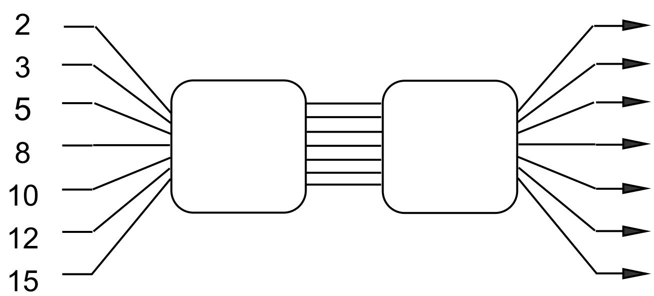
h:



k:



m:



SCARCE METAL

Use your calculator in this activity where needed.
Share the work in your group.

1. The cost, in rands, of x g of a scarce metal is given by the formula

$$C = \frac{x^2 - 4}{x - 2}$$

Complete the table:

Mass (x g)	10	11	12	13	14	
Cost (R)						442

2. The cost of a different metal is given by

C = $x - 2$

Complete the table:

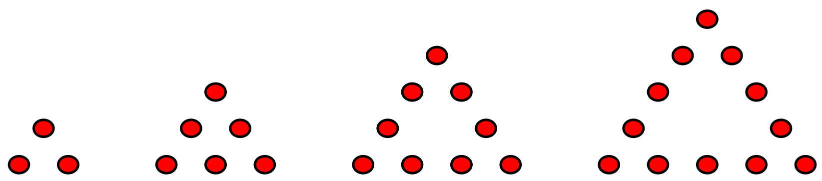
Mass (x g)	10	11	12	13	14	
Cost (R)						442

3. (a) Find the value of $\frac{x^2 - 4}{x - 2}$ if $x = 42{,}17$

(b) Solve for x if $\frac{x^2 - 4}{x - 2} = 100$

GEOMETRIC PATTERNS

1. (a) Describe in words how this growing pattern of triangles is made.



Triangle 1 Triangle 2 Triangle 3 Triangle 4

(b) Describe Triangle 6 and Triangle 7 in words. How many dots are there in Triangle 6 and how many in Triangle 7?

(c) Describe Triangle 60 and Triangle 70 in words. Calculate the number of dots in Triangle 60 and in Triangle 70.

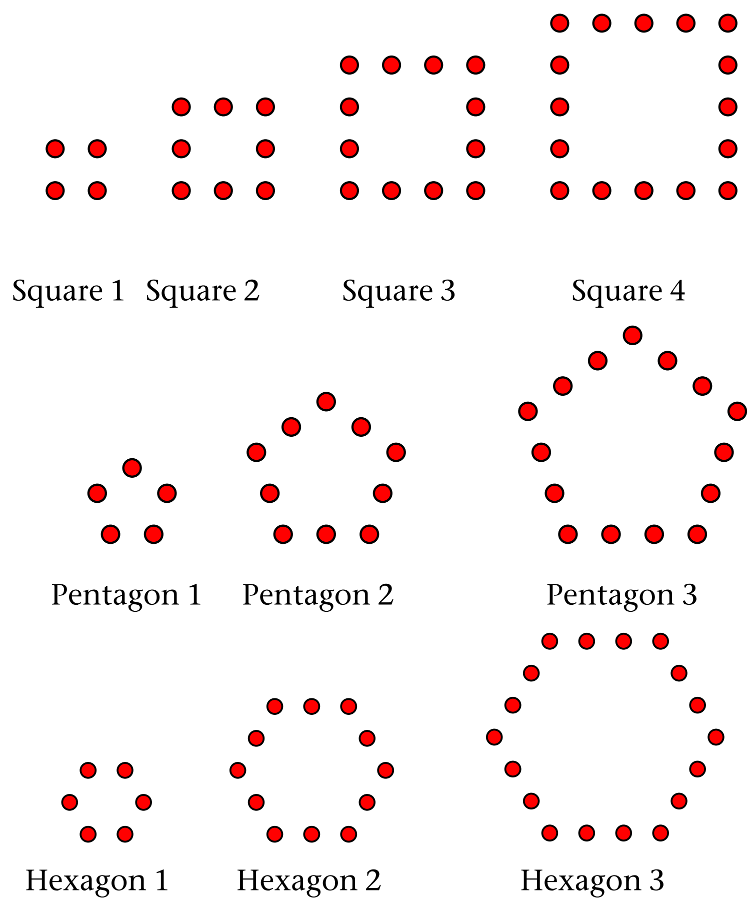
(d) Complete this table. Describe and discuss your methods. Describe and discuss what patterns you see in the table.

Triangle number	1	2	3	4	5	6	30
Number of dots	3	6					

2. On the next page you can see growing patterns of squares, pentagons and hexagons. Investigate each pattern by answering these questions.

(a) Describe in words how the growing pattern is made.

(b) Describe Figure 6 (that is Square 6, Pentagon 6 and Hexagon 6) and Figure 7 in words. How many dots are there in Figure 6 and how many in Figure 7?



(c) Describe Figure 60 and Figure 70 in words. Calculate the number of dots in Figure 60 and in Figure 70.

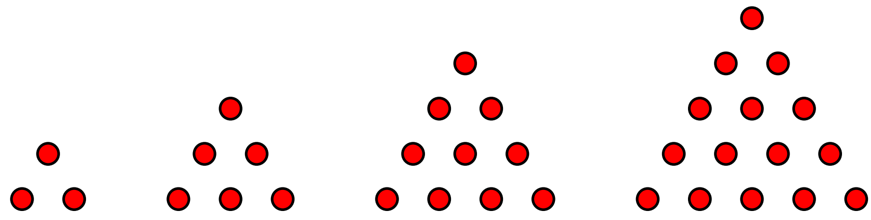
(d) Complete this table. Describe and discuss your methods. Describe and discuss what patterns you see in the table.

Figure number	1	2	3	4	5	6	30
No. of dots in square	4	8					
No. of dots in pentagon	5	10					
No. of dots in hexagon	6						

(e) How are the patterns of triangles, squares, pentagons and hexagons the same, and how are they different?

More dot patterns

1. (a) Describe in words how this growing pattern of triangles is made.



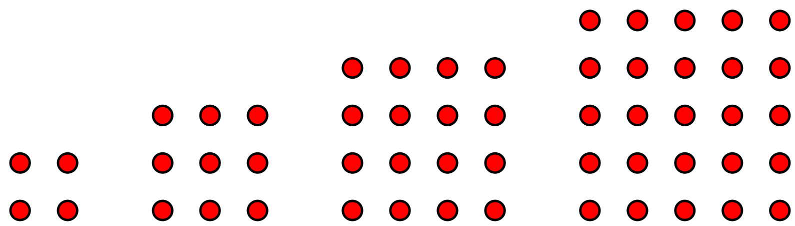
Triangle 1 Triangle 2 Triangle 3 Triangle 4

(b) Describe Triangle 6 and Triangle 7 in words. How many dots are there in Triangle 6 and how many in Triangle 7?

(c) Complete this table. Describe and discuss your methods. Describe and discuss what patterns you see in the table.

Triangle no.	1	2	3	4	5	6	7	8	9	10
No. of dots	3	6								

2. (a) Describe in words how this growing pattern of squares is made.



Square 1 Square 2 Square 3 Square 4

(b) Complete this table. Describe and discuss your methods. Describe and discuss what patterns you see in the table.

Square no.	1	2	3	4	5	6	7	8	30
No. of dots	4	9							

Tile patterns

1. Lindiwe is making this growing pattern of pictures with squares:

Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

If Lindiwe continues the pattern:

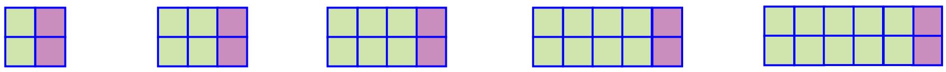
(a) Describe in words what Picture 6 and Picture 7 will look like.
Draw Picture 6 and Picture 7.
How many squares are there in Picture 6 and how many in Picture 7?

(b) Describe in words what Picture 60 and Picture 70 will look like.
Do not draw them! Imagine them, "see" them in your head!.
Calculate the number of squares in Picture 60 and in Picture 70.

2. Simphiwe is making these growing patterns of pictures with squares.

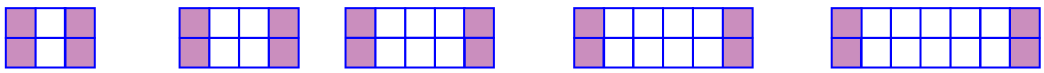
For each of these patterns, answer the same questions (a) and (b) as above.

(a)



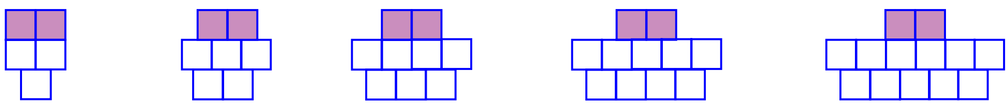
Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

(b)



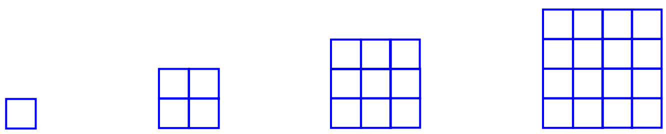
Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

(c)



Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

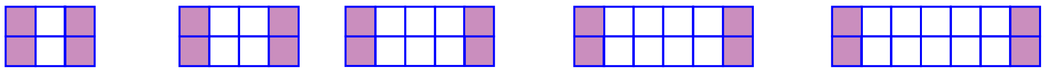
(d)



Picture 1 Picture 2 Picture 3 Picture 4

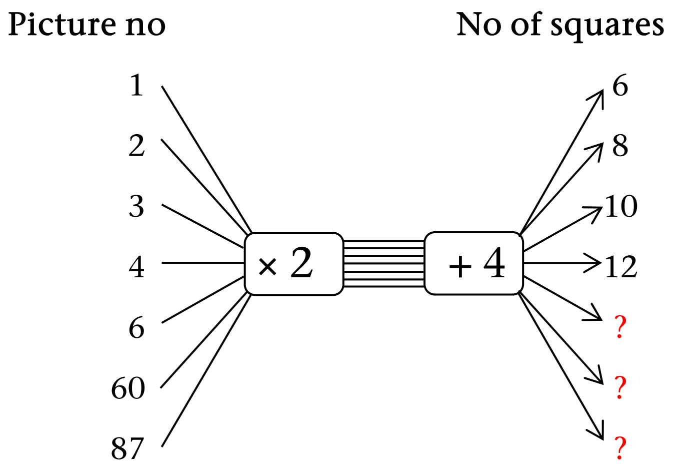
Writing our plans as flow diagrams

Let us now return to this previous problem and look at it differently. What is your plan to calculate the number of squares in Picture 6, Picture 60 and Picture 87?



Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

Mary writes her plan as this flow diagram:



A **flow diagram** describes a plan (method) using **input → rule → output**

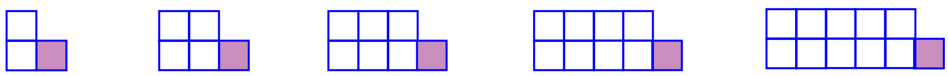
This flow diagram shows that you must first multiply any **input number** by 2 and then add 4 to get the **output number**.

3×2 → **6; 6 + 4 → 10**

4×2 → **8; 8 + 4 → 12**

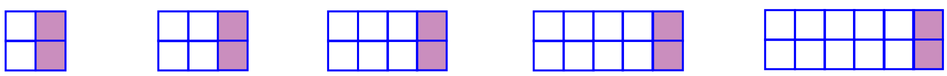
1. First check if Mary's flow diagram (plan) is correct for the known input and output numbers. Discuss how to check her plan.
2. If Mary's plan is correct, use her plan to calculate the missing output numbers.
3. Mary says with the flow diagram it is easy to calculate the number of squares in Picture 60 or in Picture 87 or in any Picture number. Do you agree?
4. If you or your friends have a different plan than Mary, write it as a flow diagram.
5. Write your plans for each of these patterns as flow diagrams, and calculate the number of squares in Picture 6, Picture 60 and Picture 87. Compare your methods with other learners' methods.

(a)



Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

(b)



Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

6. Here is another triangle pattern.

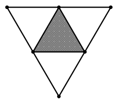


Figure 1

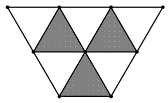


Figure 2

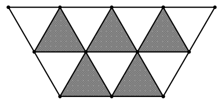


Figure 3

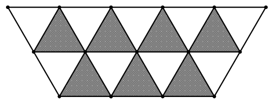
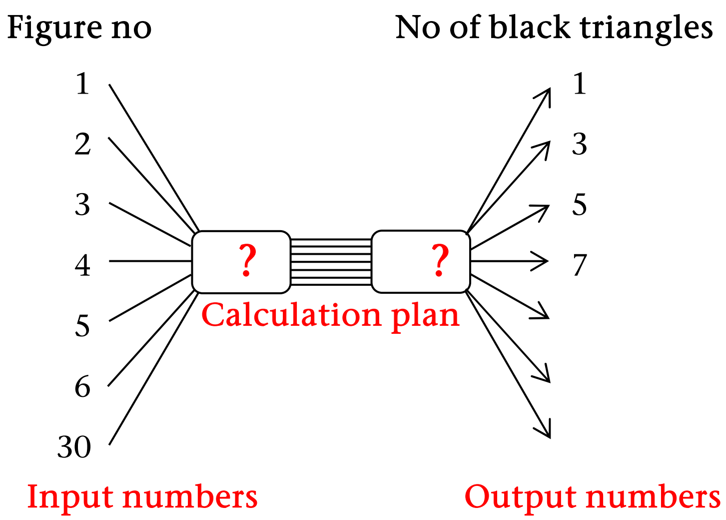


Figure 4

Complete this table. Describe and discuss your methods. Describe and discuss what patterns you see in the table.

Figure number	**1**	**2**	**3**	**4**	**5**	**6**	**30**
Number of black triangles	1	3	5	7			
Number of white triangles	3	5					
Total number of triangles	4	8					

7. (a) Make a flow diagram to show how the number of black triangles can be calculated. The Picture numbers as in the table above are the **input numbers**. Write your table answers in the flow diagram as the **output numbers.** What **calculation plan** will give the right answers? Check it.



(a) Repeat (a) for the number of white tiles, and then repeat again for the total number of tiles.

Triangle tile patterns

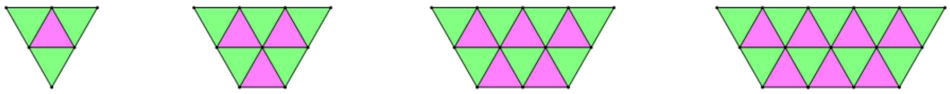
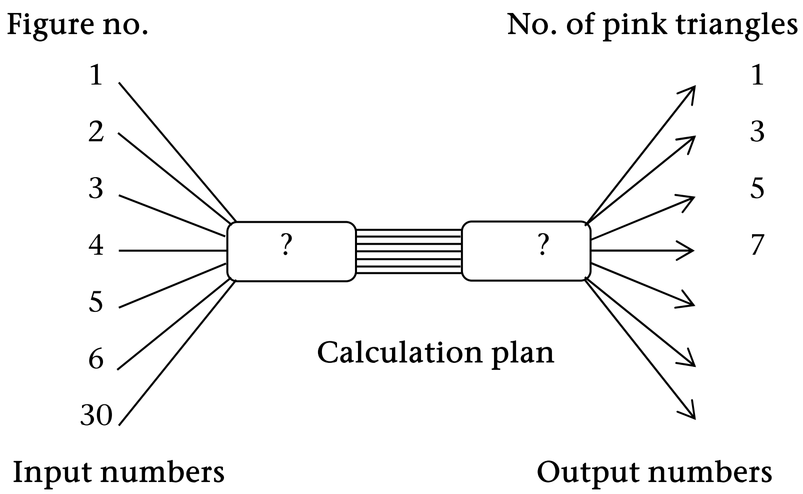


Figure 1 Figure 2 Figure 3 Figure 4

1. Complete this table. Describe and discuss your methods. Describe and discuss what patterns you see in the table.

Figure number	1	2	3	4	5	6	30
Number of pink triangles	1	3	5	7			
Number of green triangles	3	5					
Total number of triangles	4	8					

2. Make a flow diagram to show how the number of pink triangles can be calculated. Use the Figure numbers as the input numbers and the number of pink triangles as the output numbers. Complete all missing parts.

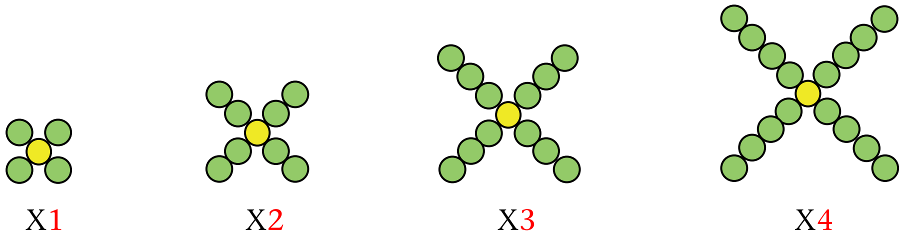


3. Repeat question 2, but this time make a flow diagram to show how the number of green triangles can be calculated.

4. Repeat question 2 again but show how the total number of triangles can be calculated.

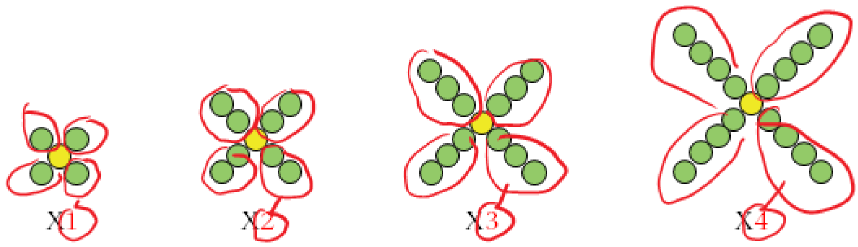
Writing calculation plans

1. Thabo uses beads to make a pattern of Xs like this:



   (a) If Thabo continues the pattern, how many beads will there be in X5, how many in X6, how many in X50 and how many in X60?

   (b) Mary uses clever counting to answer question 1! Try to follow her reasoning. Explain her plan to a classmate.



Mary starts here:

I see four, four, four greens plus one yellow

X4 = 4 × 4 + 1

← ←

Then here:

Four threes plus 1

X3 = 4 × 3 + 1

← ←

Then here:

X2 = 4 × 2 + 1

← ←

X1 = 4 × 1 + 1

X***number*** = 4 × ***number*** + 1

It means "multiply the ***number*** by 4, then add 1".

So X5 = 4 × 5 + 1

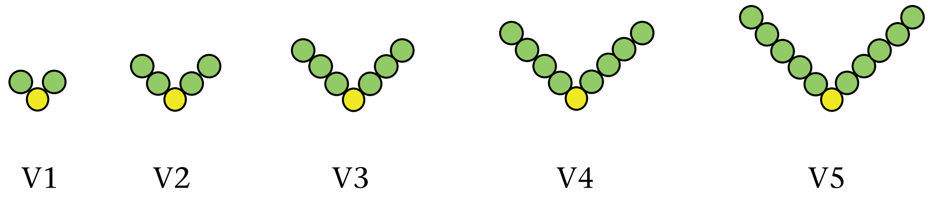
So X6 = 4 × 6 + 1

So X50 = 4 × 50 + 1

So X60 = 4 × 60 + 1

> Mary writes a **calculation plan (rule or formula)**:
>
> X*number* = 4 × *number* + 1
>
> Now she can calculate X*number* for <u>any</u> *number*.

2. Suzi uses beads to make this growing V-pattern:



(a) Describe in words what V6, V60 and V87 will look like.

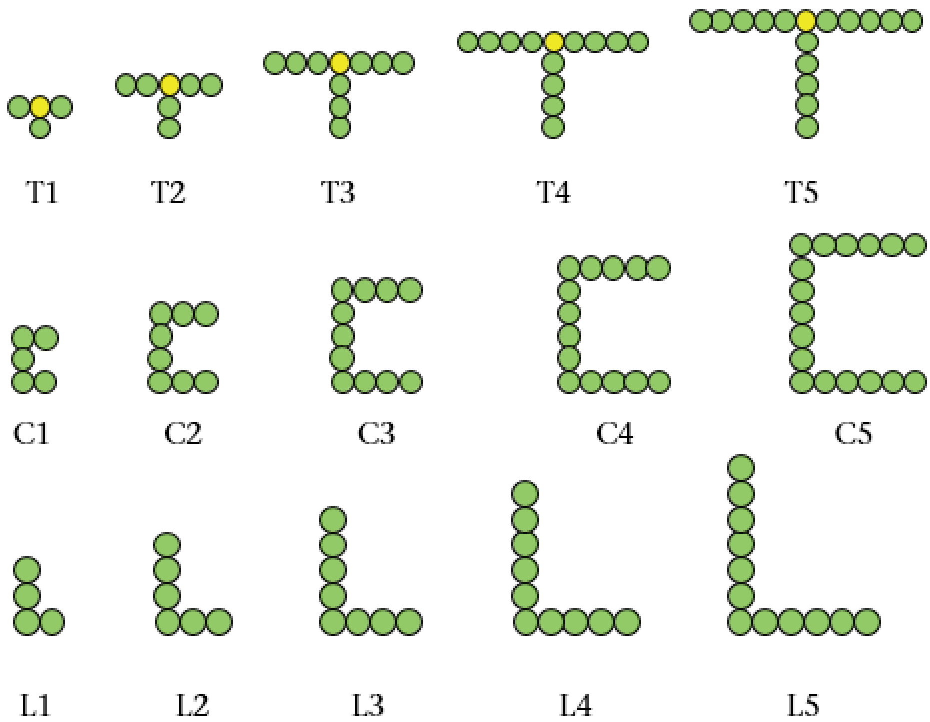
(b) Write your plan as a *flow diagram* and then calculate the number of beads in V6, V60 and V87.

(c) Write down your calculation plan, and then use it to calculate the total number of beads in V6, V60 and V87.

(d) What is the biggest V-number that can be made with 100 green beads and one yellow bead? How many beads are left over?

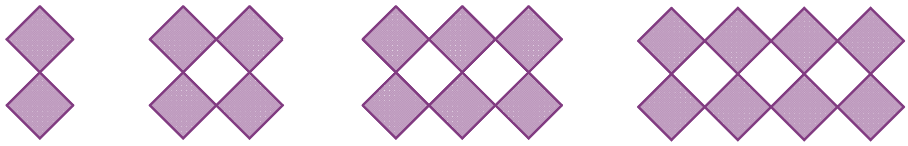
3. Sam uses beads to make these different alphabet patterns.

Answer the same questions as in question 2 for these T, C and L patterns.



4.3 Describing patterns

Purple tiles and white tiles are arranged to make this growing pattern:



Size 1 Size 2 Size 3 Size 4

1. Complete the table. Describe your methods.

Size	1	2	3	4	5	6	30
No. of purple tiles	2	4	6				
No. of white tiles	0	1	2				
Total no. of tiles	2	5	8				

2. Describe horizontal numeric patterns for the purple titles, for the white tiles and for the total number of tiles in the table.

   How can you use these horizontal patterns to calculate the number of purple tiles, the number of white tiles and the total number of tiles?

Horizontal means from left to right; **vertical** means from top to bottom.

3. Describe vertical numeric patterns for the purple tiles, for the white tiles and for the total number of tiles in the table.

   How can you use these patterns to calculate the number of purple tiles, the number of white tiles and the total number of tiles?

4. How many purple tiles are there in a Size 50 pattern?

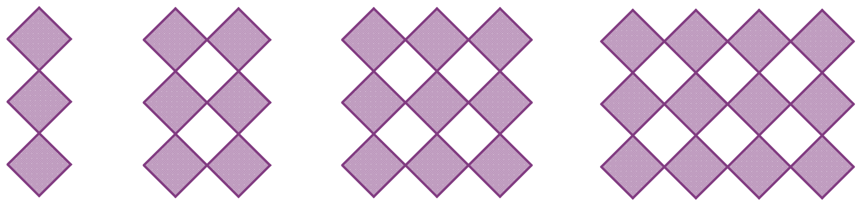
5. How many white tiles are there in a Size 50 pattern?

6. How many tiles are there in total in a Size 50 pattern?

7. Here are three other growing geometric patterns made with purple and white tiles.

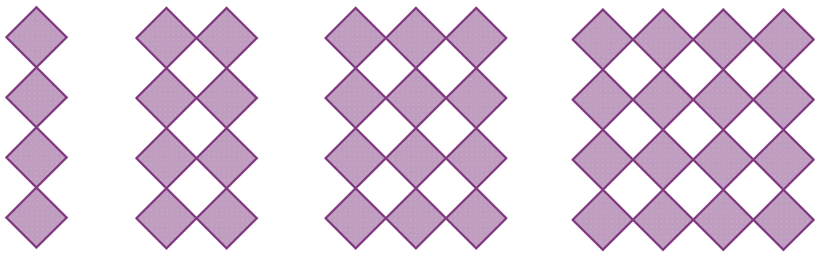
Look at them carefully. Then answer the same questions as in questions 1 to 6 for each tile pattern.

(a) Pattern X



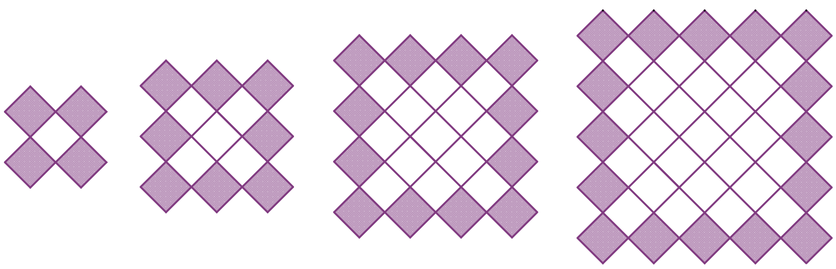
Size 1 Size 2 Size 3 Size 4

(b) Pattern Y



Size 1 Size 2 Size 3 Size 4

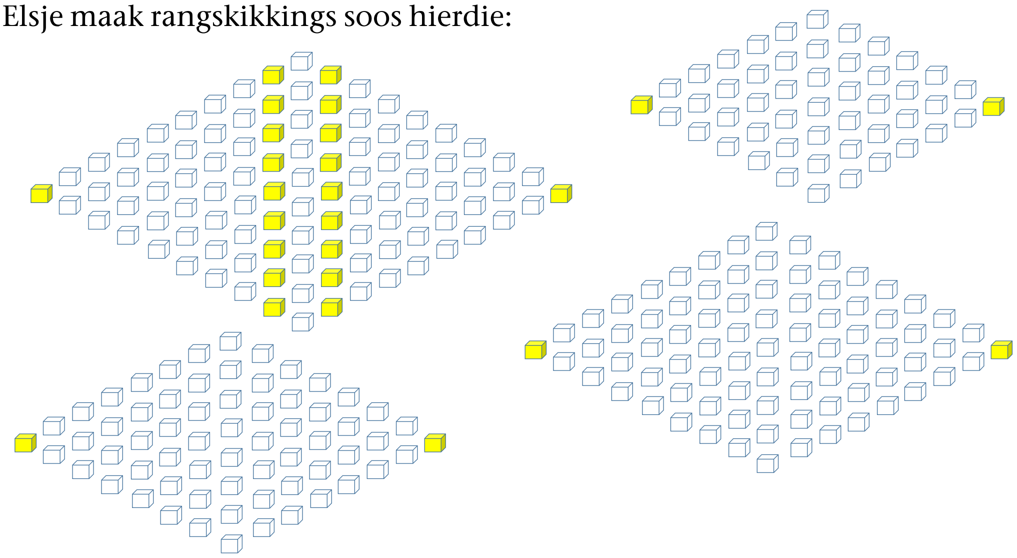
(c) Pattern Z



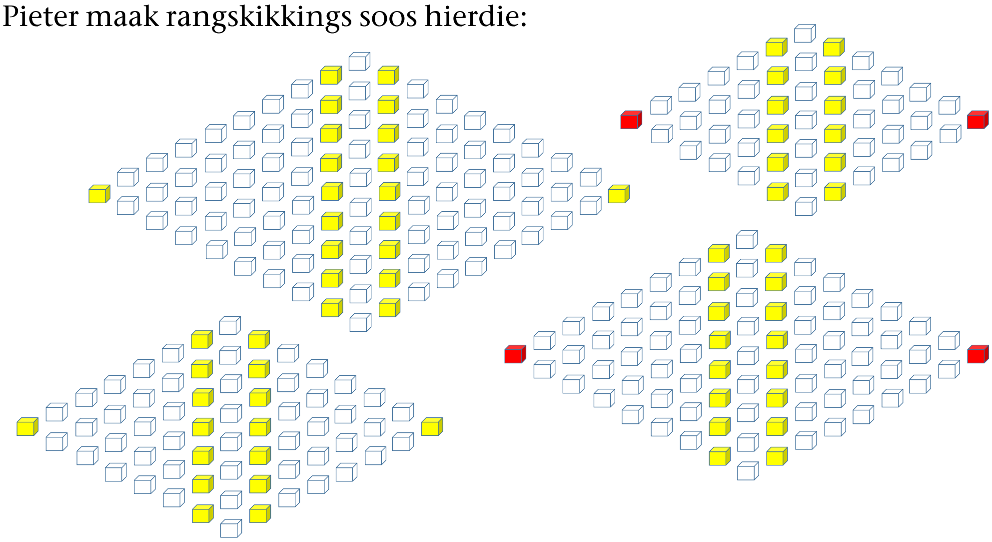
Size 1 Size 2 Size 3 Size 4

18. Kyk mooi vir verskille

Elsje maak rangskikkings soos hierdie:



Pieter maak rangskikkings soos hierdie:



1. Hoeveel geel blokkies sal Pieter en Elsje elkeen nodig hê vir hulle rangksikkings wat 19 blou blokkies bevat?

   Skryf 'n netjiese verslag van wat jy gedink en gedoen het om by jou antwoorde uit te kom.

2. Hoeveel geel blokkies sal Pieter en Elsje elkeen nodig hê vir hulle rangksikkings wat 71 blou blokkies bevat?

19. Getalrye

1. Skryf die volgende drie getalle in elke ry neer, en beskryf hoekom jy dink dit die getalle moet wees.

(a)	17	21	25	29	33	...	...	...
(b)	2	10	50	250	1 250	...	...	...
(c)	2	5	10	18	29	...	...	...

As jy dele van vraag 2 baie moeilik vind kan jy eers vraag 3 doen, en selfs vraag 4 ook.

2. (a) Watter getalry hier onder is gevorm deur by 5 te begin en herhaaldelik 10 by te tel?
   (b) Watter getalry is gevorm deur by 5 te begin en herhaaldelik met 3 te maal?
   (c) Watter getalry is gevorm deur by 5 te begin en dan om die beurt 10 en 5 by te tel?
   (d) Watter getalry is gevorm deur by 5 te begin, 10 by tel en dan elke keer 10 meer as die vorige keer by te tel?

Getalry A:	5	15	35	65	105	155	215
Getalry B:	5	15	30	50	75	105	140
Getalry C:	5	15	25	35	45	55	65
Getalry D:	5	15	45	135	405	1 215	3 645
Getalry E:	5	15	20	30	35	45	50

3. Skryf die volgende drie getalle in elkeen van die rye hier bo neer, en beskryf die berekeninge wat jy gedoen het.

Om te probeer uitpluis hoe ’n getalry gevorm is, kan ’n mens die verskille tussen opeenvolgende getalle bereken en neerskryf soos hier vir getalry A gewys word:

Die terme van die ry:	5	15	35	65	105	155	215
Verskille tussen terme:	10	20	30	40	50	60	

Daar is ’n patroon in die verskille!

4. Bereken die verskille tussen opeenvolgende terme van die rye B, C, D en E, en skryf in elke geval die ry getalle en die verskille neer soos in die voorbeeld vir getalryA gewys is.

5. Beskryf in woorde hoe getalry B gevorm is.

6. Stel vas en beskryf hoe elkeen van hierdie getalrye gevorm is.

Getalry F:	43	62	81	100	119	138	
Getalry G:	13	18	26	37	51	68	88
Getalry H:	43	38	33	28	23	18	13
Getalry I:	28	36	44	52	60	68	76

Die **eerste term** van getaly F is 43.

Die **vierde term** van getalry F is 100.

Die **verskil** tussen die 5de en 6de terme van getralry F is 19.

81 en 100 is opeenvolgende terme van getalry F.

Om die nommers van die terme duidelik aan te dui, kan ’n mens ’n getalry in ’n tabel skryf soos hier vir getalry F:

termnommer	1	2	3	4	5	6	7	8	9	10
termwaarde	43	62	81	100	119	138				

7. (a) Wat is die eerste term van getalry H hier bo?
   (b) Wat is die derde term van getalry I?
   (c) Wat is die tiende term van getalry F?
   (d) Wat is die verskil tussen die 3de en 4de terme van die getalry G?
   (e) Skryf enige drie opeenvolgende terme van getalry H neer.

8. (a) Maak en voltooi ’n tabel soos hierdie.

termnommer	1	2	3	4	5	6
48 – (5 × die termnommer)	...	...	...	...	...	...

   (b) Watter getalry in vraag 1 se terme kan met die rekenplan 48 – (5 × *die termnommer*) bereken word?

9. Die getalry Q se terme kan met die rekenplan *4 × die termnommer + 5* bereken word. Bereken die derde, agste en veertiende terme van die getalry Q.

20. Konstante verskille tussen opeenvolgende terme

Vrae 1 tot 4 handel oor hierdie vier getalrye:

Getalry A:	15	18	21	24	27	30	33
Getalry B:	20	25	30	35	40	45	50
Getalry C:	4	11	18	25	32	39	46
Getalry D	12	18	24	30	36	42	48

1. Skryf die sesde term van elk van die rye A, B, C en D neer.

As jy wil kan jy tabelle met termnommers en termwaardes maak om dit makliker te maak om vrae oor rye A, B, C en D te beantwoord.

2. Skryf die 8ste, 9de, 10de en 11de terme van elkeen van die vier rye neer.

3. Wat is die verskil tussen opeenvolgende terme van

   (a) ry A (b) ry B (c) ry C (d) ry D?

4. Ondersoek watter van hierdie rekenplanne gebruik kan word om die terme van getalry B te bereken. (Daar kan dalk meer as een wees.)

   Rekenplan I: 3 × (*die termnommer* + 4)
   Rekenplan II: 10 × *die termnommer* + 10
   Rekenplan III: 3 × *die termnommer* + 12
   Rekenplan IV: 5 × (*die termnommer* + 3)
   Rekenplan V: 5 × *die termnommer* + 3
   Rekenplan VI: 5 × *die termnommer* + 12
   Rekenplan VII: 5 × *die termnommer* + 15
   Rekenplan VIII: 15 × *die termnommer* – 2
   Rekenplan IX: 7 × *die termnommer* – 3
   Rekenplan X: 6 × *die termnommer* + 6
   Rekenplan XI: 6 × (*die termnommer* + 1)

5. Stel vas met watter rekenplanne die terme van rye A, C en D bereken kan word.

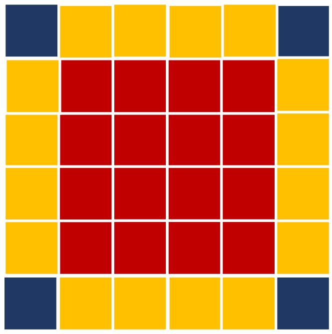
6. Probeer 'n rekenplan maak waarmee die getalry
   12 16 20 24 28 32 36 40 ..... bereken kan word.

7. Moenie nou enige berekeninge met die rekennplanne hier onder uitvoer nie. In watter gevalle ***dink jy*** die twee rekenplanne sal dieselfde getalry oplewer as dit op die getalle 1.. 2 3 4 5 6 .....10 toegepas word? Onthou dat hakies gebruik word om aan te dui dat die berekening binne die hakies eerste gedoen moet word.

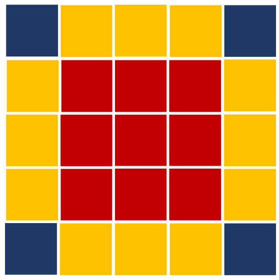
   (a) 3 × *die getal* + 6 en 3 × (*die getal* + 6)
   (b) 3 × *die getal* + 6 en 3 × (*die getal* + 2)
   (c) 3 × *die getal* – 6 en 3 × (*die getal* – 6)

21. Ontwerpe

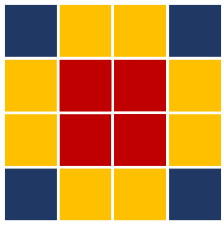
Ontwerpreeks A



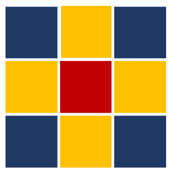
Ontwerp 4



Ontwerp 3



Ontwerp 2



Ontwerp 1

1. Ontwerpe 5, 6 en 7 moet volgens dieselfde patroon as ontwerpe 1 tot 4 gemaak word.

   Hoeveel blokkies van elke kleur is daar vir elkeen van ontwerpe 5, 6 en 7 nodig?

2. Verduidelik hoe jy gedink het, en wys watter berekeninge jy gedoen het, om die getalle blou, geel en rooi blokkies vir ontwerp 5 te bereken.

3. Hoeveel blokkies van elke kleur is vir ontwerp 10 nodig?

Die getal rooi blokkies verander van die een ontwerp na die ander. Só 'n hoeveelheid word 'n **veranderlike** genoem.

In elkeen van hierdie ontwerpe is daar 4 blou blokkies. Die getal blou blokkies verander nie van die een ontwerp na die ander nie. Ons sê die getal blou blokkies in hierdie ontwerpe is 'n **konstante**.

4. Is die getal geel blokkies in hierdie reeks ontwerpe 'n konstante of 'n veranderlike?

5. Watter hoeveelhede is veranderlikes, en watter is konstant, in ontwerpreeks B op die volgende bladsy?

6. In een van die ontwerpe in ontwerpreeks A, wat nogal groot is en nie hier gewys word nie,
   is daar 400 rooi blokkies.
   Hoeveel geel blokkies is daar in hierdie ontwerp?

7. In 'n ander een van hierdie ontwerpe is daar
   400 geel blokkies.
   Hoeveel rooi blokkies is daar in hierdie ontwerp?

8. Maak tabelle soos hierdie vir ontwerpreeks A, en voltooi dit.

Tabel 1

Ontwerpnommer	1	2	3	4	5	6	7	8	9
Totale getal blokkies									
Getal blou blokkies									
Getal rooi blokkies									
Getal geel blokkies									

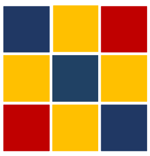
Tabel 2

Ontwerpnommer	10	20	30	40	50	60	70	80	90
Totale getal blokkies									
Getal blou blokkies									
Getal rooi blokkies									
Getal geel blokkies									

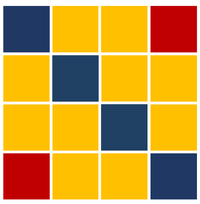
9. Wat is die nommer van die ontwerp in reeks A wat 324 geel blokkies het?

10. Wat is die nommer van die ontwerp wat 225 rooi blokkies het?

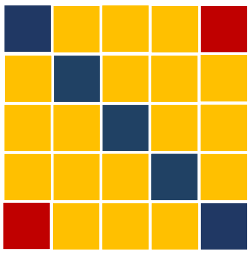
11. Is daar net soveel blokkies van elke kleur in ontwerp 21 van reeks A, as wat daar in ontwerp 20 en ontwerp 1 saam is? Maak seker dat jy reg is.



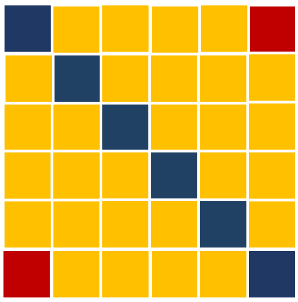
Ontwerp 1



Ontwerp 2



Ontwerp 3



Ontwerp 4

Ontwerpreeks B

12. Hoeveel geel blokkies sal daar in ontwerp 10 van reeks B wees, en hoeveel in ontwerp 20?

13. (a) Hoeveel blokkies sal daar altesaam in ontwerp 15 van reeks B wees?
    (b) Hoeveel rooi blokkies sal daar in ontwerp 15 wees?
    (c) Hoeveel blou blokkies sal daar in ontwerp 15 wees?
    (d) Hoeveel geel blokkies sal daar in ontwerp 15 wees?

14. In een van die ontwerpe in reeks B, is daar 340 geel blokkies.
    Hoeveel blou blokkies is daar in hierdie ontwerp?

15. Altesaam 34 vierkantige teëlvloere, elk met 64 teëls, moet in 'n gebou gelê word.

    (a) Hoeveel teëls van elke kleur sal nodig wees as 'n ontwerp uit reeks A gebruik word?
    (b) Hoeveel teëls van elke kleur sal nodig wees as 'n ontwerp uit reeks B gebruik word?

Rooi vloerteëls kos R3,74 elk, geel vloerteëls kos R2,93 elk en blou vloerteëls kos R4,13 elk. 'n Bouer vra 64c per teël vir die arbeid om vloere te lê, en die materiaal om die teëls te lê kos 28 c per teël.

Jy kan 'n sakrekenaar gebruik om die kosteberekeninge hier onder te doen.

16. Bereken die totale koste, volgens die pryse hier bo, vir elk van die volgende bouprojekte.

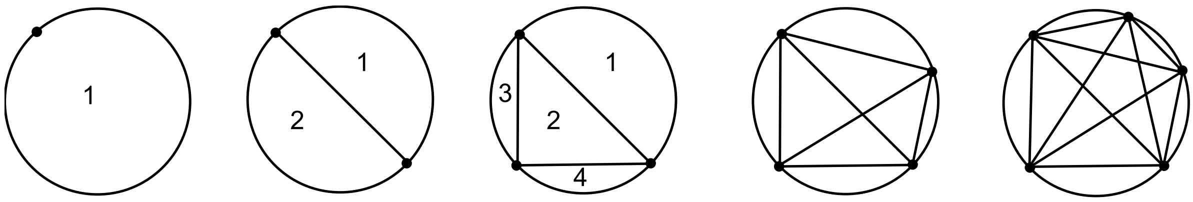
    (a) 87 vloere almal volgens ontwerp 18 in reeks A.
    (b) 136 vloere almal volgens ontwerp 24 in reeks B.
    (c) Een vloer van elkeen van ontwerpe 1 tot 12 van reeks A.
    (d) 17 vloere volgens ontwerp 18 van reeks A en 23 vloere volgens ontwerp 26 van reeks B.

17. Hoeveel geel teëls is daar altesaam in
    (a) ontwerpe 1 en 2 van reeks A
    (b) ontwerpe 1, 2 én 3
    (c) ontwerpe 1 tot 4
    (d) ontwerpe 1 tot 5
    (e) ontwerpe 1 tot 6
    (f) ontwerpe 1 tot 7
    (g) ontwerpe 1 tot 10
    (h) ontwerpe 1 tot 30

    Dit kan nuttig wees om nadat jy (a) tot (f) gedoen het die resultate in 'n tabel op te skryf, en te kyk of daar nie 'n patroon is wat vrae (g) en (h) makliker sal maak nie.

GENERALISING AND PROVING

1. REGIONS



If 3 *points* on a circle are joined, 4 *regions* are formed, as shown above.
Complete the table for 4 and 5 points by using the given sketches above.
What is the *maximum* number of regions into which 6 points on a circle will divide the circle if the points are joined? And 20 points?

# points (*p*)	1	2	3	4	5	6	20
# regions (*R*)	1	2	4	8	16		

2. ONES

Check these patterns:

x	x^2	Digit sum of x^2
1	1	1
11	121	4
111	12321	9
1111	1234321	16
11111	123454321	25

Now predict the digit sum of 11111111111^2 (11 ones squared).

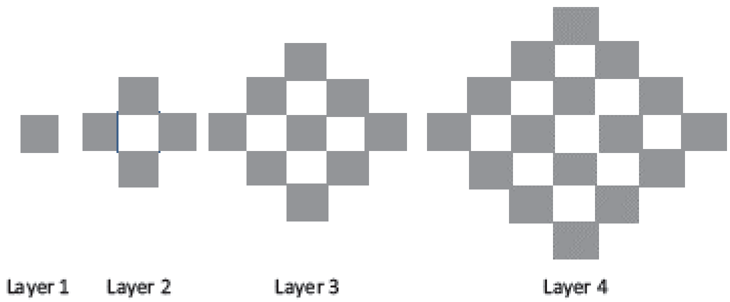
3. ONES AGAIN

Now check this:

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 10^2$
$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 11^2$

Generalise! Explain! Prove!

Can you *see* the problem here:



4. PRIMES

Investigate the nature of $P(n) = n^2 - n + 11,\ n \in N$

Let's approach the problem inductively by generating the following special cases:

$P(1) = 11$

$P(2) = 13$

$P(3) = 17$

$P(4) = 23$

$P(5) = 31$

$P(6) = 41$

$P(7) = 53$

$P(8) = 67$

$P(9) = 83$

$P(10) = 101$

> These numbers are all prime. Would you agree that we can conclude that the answer is *always* a prime number?

5. ODD NUMBERS

The examples above of numbers *P*(n) in Problem 4 are all odd. Would you agree that we can conclude that the answer is *always* an odd number? How can you be *sure?*

6. IQ PROBLEM

What is the next number in this sequence: 2; 4; 6; 8; ?

We would probably all opt for 10. Are you *sure?*

7. CONSECUTIVE NUMBERS

Investigate the sum of any three consecutive numbers ….

8. CONSECUTIVE NUMBERS

Find all the numbers that have an odd number of factors …

9. REMAINDER

Investigate the remainder when the square of an odd number is divided by 8. Are you *sure*?

10. THINK OF A NUMBER

Think of any number Double it. Add 6. Halve the result. Subtract your original number. What is your result?

Mary says: "No matter with what number you start, the final answer will always be 3".

Is she right? How do you *know?*

Can you explain *why* it is so?

11. THINK OF A NUMBER

Think of any secret number Multiply by 4. Add 6. Halve the result. Subtract your original number. What is your result?